

# Correlation Between Cone-Excitation Ratios Invariance And Perception Of Transparency

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## Abstract

It has been demonstrated by a Monte Carlo simulation [1] that cone-excitation ratios for opaque surfaces and surfaces seen through a transparent filter are almost invariant under certain conditions. We tested whether the invariance is correlated with the perception of transparency by measuring psychophysical performance for a transparency perception task in which filters producing a perfect invariance were compared with those filters whose cone-excitation ratios were not invariant. A computer controlled colour monitor was used for reproducing the chromaticities and luminances of a Mondrian of Munsell colour samples under D65 illuminant. In a 2AFC paradigm observers were asked which of the two filters appeared to be a uniform transparent filter over opaque surfaces. The results show the preference for the filter producing the highest invariance of cone-excitation ratios.

## 1. Introduction

Colour appearance is important visual information that allows us to identify objects. We are able to recognise an object even if its position has changed or it is seen under a different illuminant, or if it is partially covered by a transparent layer. An object's colour constancy can be defined as the maintenance of colour appearance despite variations in the colour of nearby objects and despite variations in the spectral power distribution of the ambient light [2].

A number of algorithms have been proposed for the purpose of modelling human colour constancy [3-11], but the underlying mechanism is still unknown. The major problem in finding the colour constancy mechanism arises because the light in the visual image confounds two factors: the spectral power distribution of the ambient light and the surface reflectance of the objects in the scene.

If there is only one unknown object illuminated by an unknown illuminant the problem has an infinite number of mathematical solutions. The solution is unique if the unknown object is not the only object in the image.

As a consequence, all the algorithms must use information obtained from light reflected from different objects in the scene.

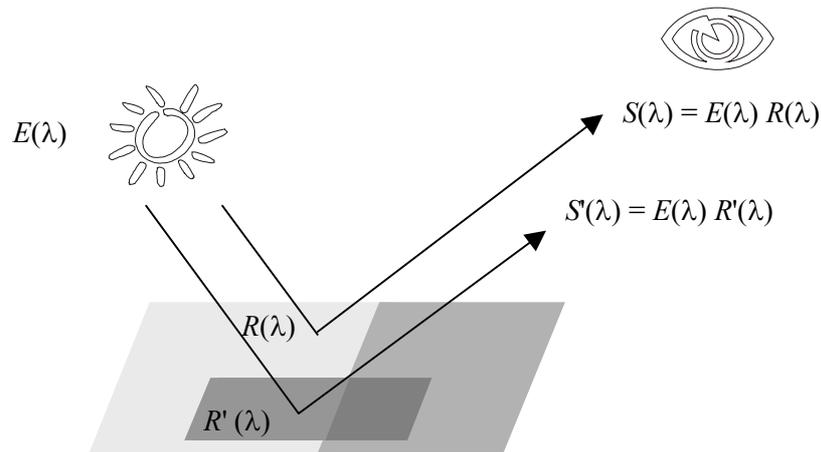
Among the models [3-7] proposed in the recent years, one has captured our attention since it can also be applied to the perception of transparency. The model states that cone-excitation ratios of pairs of surfaces seen under an unknown illuminant remain almost invariant when the same surfaces are viewed under a different illuminant. By analogy we state that cone-excitation ratios of pairs of opaque surfaces seen directly are invariant when the same surfaces are covered by a transparent filter.

## 2. Colour transparency

The light reaching the eye when an illuminant hits a surface is called the colour signal  $S$ . The spectral power distribution of  $S$  is determined by the spectral power distribution of the illuminant  $E$  and the spectral reflectance function  $R$  of the surface. The colour signal is computed by multiplying the illuminant at each wavelength by the corresponding value of the surface reflectance function. If the surface is partially covered by a transparent filter (Figure 1), the colour signal  $S'$  from the filtered area is now different from the rest of the surface seen directly. The filtered reflectance  $R'$  of the opaque surface is given by the product of the spectral reflectance function  $R$  of the surface and the transmittance  $T$  of the filter.

Despite changes in colour signal, under certain constraints the visual system is able to recognise the surface seen directly and the surface seen under the filter as a single surface. Those constraints refer to the smoothness of both the reflectance functions and the energy power distribution of the illuminant.

By performing a Monte Carlo simulation [1] we observed that if those constraints are preserved cone-excitation ratios between surfaces seen directly and the same surfaces seen under a filter are almost statistically invariant for a large set of surfaces. Whether this result is correlated to our perception of transparency is the object of this paper.



**Figure 1.** Colour signals for an opaque surface seen directly and under a transparent filter.  $R(\lambda)$  is the reflectance of the opaque surface,  $R'(\lambda)$  is the surface effective reflectance when it is covered by a transparent filter.

### 3. Cone-excitation ratios approach

Figure 2 illustrates two opaque surfaces ( $e_{i,1}$  and  $e_{i,2}$ ) covered by a simulated transparent filter.

The cone excitation  $e_{i,j}$  of cone class  $i$  (where  $i \in \{L, M, S\}$  denoting long-, medium, and short-wavelength-sensitive cone classes) for a surface  $j$  seen directly is given by the integration of the product between the surface reflectance  $R(\lambda)$ , the illuminant  $E(\lambda)$ , and the cone sensitivity functions  $\phi_i$ . Thus

$$e_{i,j} = \int E(\lambda) R(\lambda) \phi_i(\lambda) \delta\lambda . \quad (1)$$

The cone excitation  $e'_{ij}$  for that surface covered by a filter is given by a more complex function [12, 13] which takes into account the transmittance of the filter. The effective reflectance  $R'$  of the filtered area is now equal to

$$R'(\lambda) = R(\lambda)[T(\lambda) (1-r)^2]^2 \quad (2)$$

where  $r$  is the internal reflectance of the filter,  $T$  is the transmittance defined by  $T(\lambda) = c$ ; where  $c$  is selected within the range of  $0 \leq c \leq 1$  for achromatic filters and by a Gaussian distribution of the form

$$T(\lambda) = 0.4 + 0.6 \{ \exp [ - (\lambda - \lambda_m)^2 / 2 \sigma^2 ] \} \quad (3)$$

where  $400\text{nm} \leq \lambda_m \leq 700\text{nm}$ ,  $5\text{nm} \leq \lambda_m \leq 200\text{nm}$  for chromatic filters.

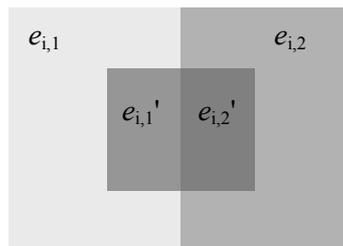
The cone-excitations for the filtered surface are equal to

$$e'_{ij} = \int E(\lambda) R(\lambda) [T(\lambda) (1 - r)^2]^2 \phi_i(\lambda) \delta\lambda . \quad (4)$$

The principle of invariance of cone-excitation ratios states that the ratio between two opaque surfaces and the ratio between the same surfaces covered by a filter is almost statistically invariant. This can be expressed by the equation

$$e_{i1}/e_{i2} = e'_{i1}/e'_{i2} . \quad (5)$$

In our previous work we tested this hypothesis by simulating opaque surfaces randomly selected. A homogeneous transparent filter was simulated according to Equation 2, and the cone-excitations for the surfaces seen directly and for the surfaces seen under the filter were calculated. We found that the invariance is weakest when the transmittance of the filter is low at some or all wavelengths, such as with narrowband chromatic filters, whereas the invariance is strong for filters that allow substantial transmittance in most of the visible spectrum. The ratios are always invariant for achromatic filters since the term  $T$  in Equation 4 is a scalar and simply cancels out.



**Figure 2.** Two opaque surfaces covered by a homogeneous transparent filter.  $e_{i1}$ ,  $e_{i2}$ ,  $e'_{i1}$ , and  $e'_{i2}$  are their correspondent cone-excitation ratios.

#### 4. Psychophysical experiment

The hypothesis of a correlation between the invariance of cone-excitation ratios and the perception of transparency has been tested in a psychophysical experiment whereby filters producing different cone-excitation ratios were compared.

In particular, filters controlled by different values of  $\sigma$  (i.e. the standard deviation of the transmittance function in Equation 3) were simulated lying on the top of a Mondrian. The

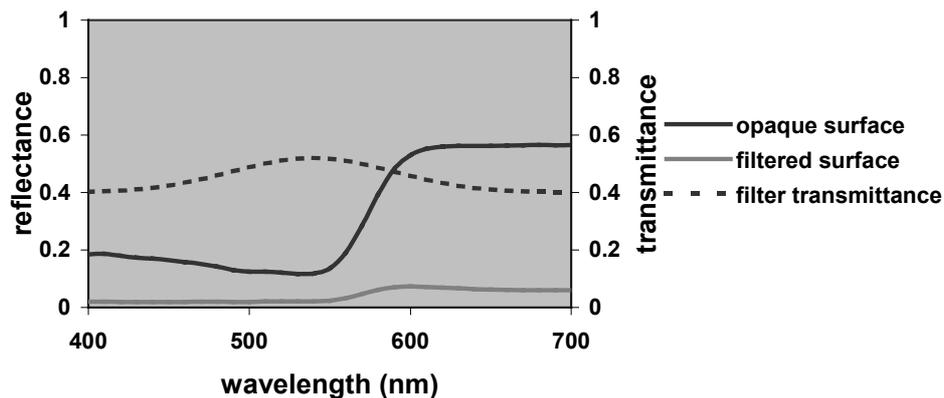
cone-excitation ratios generated by the correspondent colour signals were selected in order to produce (i) invariant cone-excitation ratios; (ii) identical cone-excitation ratios (as a control situation); (iii) cone-excitation ratios with 25% of noise; (iv) cone excitation ratios with 50% of noise. According to our model, in a discrimination task the filter generating cone-excitation ratios closer to the invariance is to be preferred.

#### 4.1 Equipment

A Sony Trinitron GMD500 colour monitor, driven by a VSG2/3 video card of a personal computer was used for presenting the stimulus pattern. The resolution was 1152 X 864 pixels and the frame rate was 120 Hz. The monitor had been calibrated and gamma corrected.

#### 4.2 Stimuli

The stimuli consisted of a Mondrian (7.5 X 7.5 deg) partially overlaid by a simulated transparent filter (2.4 X 9.5 deg) displayed at the centre of a CRT monitor. The simulated illumination was D65. The opaque surfaces composing the Mondrian were selected from 1269 samples of the *Munsell Book of Color* [14]. A pre-selection was made in order to choose only those surfaces that lay in the colour gamut of the monitor and thus could accurately be reproduced even when 50% of noise was added to them.



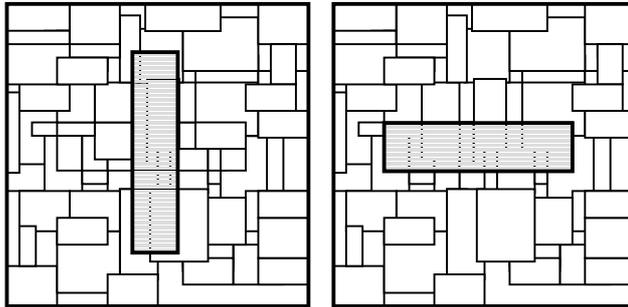
**Figure 3.** Example of a spectral reflectance function  $R(\lambda)$  for one of the opaque surfaces simulated in the experiment and its effective reflectance  $R'(\lambda)$  when it is covered by filter transmittance  $T(\lambda)$ .

The reflectance of the filtered surfaces was simulated using Equation 2. The transmittance was defined by a Gaussian distribution described in Equation 3, where  $\lambda_m$  was randomly selected in the range  $400\text{nm} \leq \lambda_m \leq 700\text{nm}$ , and  $\sigma$  could be 5nm, 25nm, or 50nm.

Figure 3 shows the spectral reflectance  $R$  of a simulated opaque surface, the transmittance function  $T$  of one of the filters used in the experiment, and the effective reflectance  $R'$  of the opaque surface after having been filtered.

In the mathematical simulation [1] we used chromatic filter transmittances described by the function  $T(\lambda) = \exp[-(\lambda - \lambda_m)^2/2\sigma^2]$  whereas in the psychophysical experiment the filter transmittance was normalised such that despite the value of  $\sigma$  it had always the same total amount of transmittance.

The internal reflectance  $r$  was set equal to 0.1 throughout all the experiment.



**Figure 4.** Simulation of two Mondrian patterns covered by a vertical (left hand side) and horizontal (right hand side) transparent filters.

### 4.3 Procedure

In a 2-alternative forced choice (2AFC) task four naïve observers viewed a simulation of a Mondrian partially overlaid by a transparent filter. In each trial a physically plausible filter was matched with one of four comparison filters. Three of those filters were a modification of the physically plausible filter and were obtained by making the short-, medium-, and long-wavelength-sensitive cone classes of the filtered colour signal invariant (indicated here as *perfect filter*), or adding two different amount of noise 25% (indicated here as *25% noise*), and 50% (indicated here as *50% noise*). The fourth comparison filter (indicated here as *real*) was almost identical to the physically plausible filter and was used as a control situation.

Each presentation lasted two seconds on screen. The next trial was not presented until the subject's answer was given.

Filters had different orientations (vertical versus horizontal) in the two intervals, and their presentation order was randomised. The vertical and horizontal filters lay over different opaque surfaces (Figure 4); thus, even when a physically plausible transparent filter was compared with a real filter with a different orientation, their cone-excitation ratios were not exactly the same.

Observers indicated whether a vertical or horizontal homogeneous transparent filter covered the Mondrian by pressing one of two buttons on a pushbutton switch box. Each trial was repeated three times and the session of 72 trials was run three times. A training of twenty trials was given before each session and subsequently discarded. No feedback was used during the experiment.

### 4.4 Results

Trials were classified according to the degree of deviation from the invariance in spatial cone-excitation ratios for all the possible pairs of surfaces seen directly and under the filter displayed in each image. The degree of deviation was calculated as follows. For any pair of surfaces seen directly and under the filter three ratios (as indicated in Equation 5) of cone-excitations can be defined: the ratio between their short-wavelength sensitive cones (for brevity  $r_s$ ); the ratio between their medium-wavelength sensitive cones ( $r_m$ ); the ratio between their long-wavelength sensitive cones ( $r_l$ ). Mean deviations from an invariant ratio (i.e. ratio equal to 1) were then calculated for each single presentation separately for each of the three cone-classes. Results are shown in Table 1 a-c.

When the comparison filter is a perfect filter its cone-excitation ratios are always equal to one and therefore its deviations are equal to zero.

When the filter is a physically plausible filter, its ratios vary according to the physical properties of the filter which in the experiment have been controlled by varying its standard deviation. As shown by the mathematical simulation [1], for low values of  $\sigma$  (i.e. for narrowband filters) cone-excitation ratios are far from invariant. As the filter becomes more and more broadband the cone-excitation ratios approximate the invariance. This also means that narrowband filters deviate more from invariant ratios respect to broadband filters. This can be seen in Tables 1b-c where deviations of real filters decrease more and more as the value of  $\sigma$  increases. An interesting result is that this does not apply to the short-wavelength sensitive cone. A possible explanation will be proposed in the discussion section.

Both the filter with 25% of noise and the filter with 50% of noise have ratios far away from invariance. As illustrated in Table 1a-c, this is accompanied by very high cone-excitation ratios deviations.

*Table 1a. Mean deviations for the two presentations in each trial for the short-wavelength sensitive cone class.*

| $\sigma$ | S cone deviations |            |        |
|----------|-------------------|------------|--------|
|          | real              | comparison |        |
| 5nm      | 0.0035            | perfect    | 0.0000 |
|          | 0.0012            | real       | 0.0038 |
|          | 0.0012            | noise 25%  | 0.1685 |
|          | 0.0044            | noise 50%  | 0.3527 |
| 25nm     | 0.0025            | perfect    | 0.0000 |
|          | 0.0043            | real       | 0.0074 |
|          | 0.0077            | noise 25%  | 0.2021 |
|          | 0.0032            | noise 50%  | 0.3773 |
| 50nm     | 0.0048            | perfect    | 0.0000 |
|          | 0.0029            | real       | 0.0023 |
|          | 0.0059            | noise 25%  | 0.1387 |
|          | 0.0027            | noise 50%  | 0.3803 |

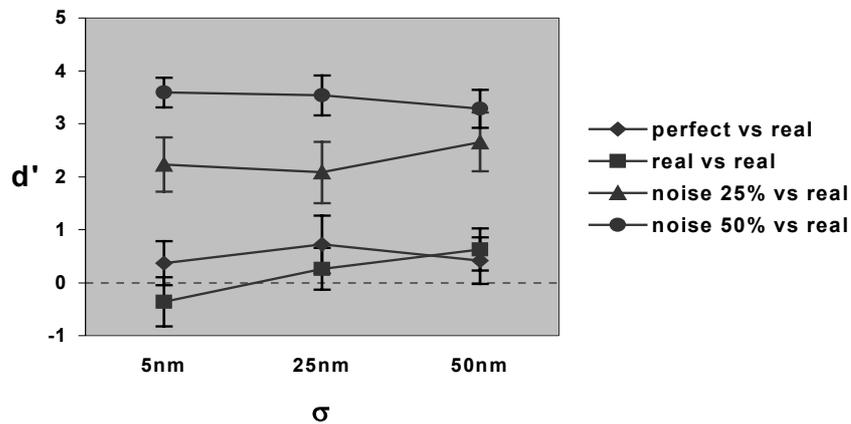
*Table 1b. Mean deviations for the two presentations in each trial for the medium-wavelength sensitive cone class.*

| $\sigma$ | M cone deviations |            |        |
|----------|-------------------|------------|--------|
|          | real              | comparison |        |
| 5nm      | 0.0787            | perfect    | 0.0000 |
|          | 0.0195            | real       | 0.1540 |
|          | 0.0077            | noise 25%  | 0.2317 |
|          | 0.0041            | noise 50%  | 0.6222 |
| 25nm     | 0.0140            | perfect    | 0.0000 |
|          | 0.0155            | real       | 0.0222 |
|          | 0.0132            | noise 25%  | 0.1299 |
|          | 0.0167            | noise 50%  | 0.6078 |
| 50nm     | 0.0053            | perfect    | 0.0000 |
|          | 0.0075            | real       | 0.0041 |
|          | 0.0035            | noise 25%  | 0.1783 |
|          | 0.0025            | noise 50%  | 0.5235 |

**Table 1c.** Mean deviations for the two presentations in each trial for the long-wavelength sensitive cone class.

| $\sigma$ | L cone deviations |            |        |
|----------|-------------------|------------|--------|
|          | real              | comparison |        |
| 5nm      | 0.0777            | perfect    | 0.0000 |
|          | 0.0777            | real       | 0.0794 |
|          | 0.04279           | noise 25%  | 0.2252 |
|          | 0.0053            | noise 50%  | 0.2231 |
| 25nm     | 0.0108            | perfect    | 0.0000 |
|          | 0.0200            | real       | 0.0300 |
|          | 0.0113            | noise 25%  | 0.1321 |
|          | 0.0363            | noise 50%  | 0.7427 |
| 50nm     | 0.004             | perfect    | 0.0000 |
|          | 0.0126            | real       | 0.0036 |
|          | 0.0041            | noise 25%  | 0.1016 |
|          | 0.0034            | noise 50%  | 0.4864 |

We tested the ability to discriminate between a physically plausible filter versus any of the comparisons by measuring d prime ( $d'$ ) values.



**Figure 5.** Mean values of  $d'$  values for the four conditions plotted against different  $\sigma$  levels.

In Figure 5, mean values of  $d'$  for all the four conditions have been plotted against  $\sigma$  levels. Four independent one-way ANOVAs have been performed in order to test the variance in each condition as the filters become more and more broadband. No significant difference has been found in the discrimination task for any of the four conditions [ $F_{2,22} = 0.21$ ,  $p = 0.81$  for the *perfect versus real* condition;  $F_{2,22} = 2.3$ ,  $p = 0.12$  for the *real versus*

real condition;  $F_{2,22} = 0.41$ ,  $p = 0.66$  for the *noise 25% versus real* condition;  $F_{2,22} = 0.24$ ,  $p = 0.79$  for the *noise 50% versus real* condition].

Since there is no effect of  $\sigma$  on performance,  $d'$  means have been calculated and plotted according to the comparison filter in each condition. As it is shown by Figure 6, there is a significant trend ( $F_{3,105} = 42.26$ ,  $p = 0.00$ ) depending on the condition. In particular, when the physically plausible filter is compared to a filter with approximately the same cone-excitation ratios deviations the performance is chance. This is the case of the *perfect versus real* condition ( $t_{35} = 1.89$ ,  $p = 0.07$ ) and the *real versus real* condition ( $t_{35} = 0.71$ ,  $p = 0.48$ ).

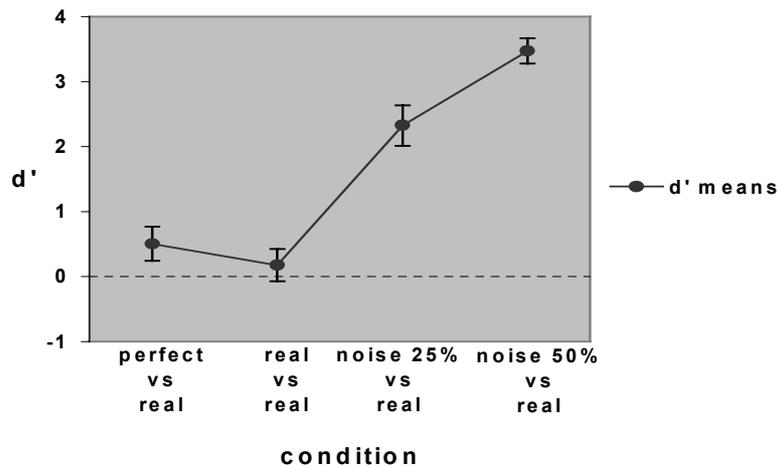


Figure 6.  $d'$  means for all the four conditions.

In the *perfect versus real filter* condition the cone-excitation ratios deviations of the physically plausible filter are not sufficiently different from the invariance to be able to distinguish which filter is a homogeneous transparent layer.

In the *real versus filter with 25% of noise* condition subjects were able to discriminate between the two filters and choose the filter with the lowest deviations from the invariance. A t Student run for this condition showed that subjects' performance was significantly different from zero ( $t_{35} = 7.48$ ,  $p = 0.00$ ).

In the *real versus filter with 50% of noise* condition  $d'$  values were even higher than the previous condition. Also in this case subjects' performance was significantly different from zero ( $t_{35} = 17.98$ ,  $p = 0.00$ ).

## 5. Discussion

The perception of transparency seems to have some analogies with colour constancy. For the colour constancy phenomenon it has been proposed that cone-excitation ratios for pairs of surfaces are almost invariant under changes in illumination and that these ratios offer a possible basis for perceptual colour constancy. In this paper we have given evidence that the invariance of cone-excitation ratios is also a possible cue for the perception of transparency.

Deviations of cone-excitation ratios from invariance have been varied in a 2AFC task where presentations having zero deviations or close to zero deviations have been compared with presentations whose deviations were significantly different from zero.

Subjects' performance is chance when zero deviations are compared. This is the case of *perfect versus real filter* condition and *real versus real filter* condition.

The performance is above chance when cone-excitation ratios between the surfaces seen directly and the surfaces under the filter give rise to deviations far from the invariance. This is the case of the filters with 25% and 50% of noise.

It is still not clear how close to the invariance the deviations have to be in order to give rise to a transparent percept. Also it is not known whether all the three cone classes need to be invariant or whether one cone class invariance is enough to perceive transparency.

Another question arising from this experiment is whether the number of surfaces present in an image matters. From our model at least two surfaces are needed in order to calculate a ratio. But no further assumptions are made about the number of surfaces.

In the real world there is likely to be more than two surfaces in a scene. Is the number of surfaces equivalent to more information for our visual system? Does our visual system perform better in this case? These are still open questions.

## 6. References

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