Recovering spectral information using digital camera systems

Coloration Technology

Society of Dyers and Colourists

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There is an increasing need to be able to measure colour properties of complex surfaces or images for which traditional spectrophotometers are not suitable. New multispectral imaging systems are being developed but it is not clearly understood how the parameters (such as the number of colour channels, the spectral properties of the channels, and the choice of illuminant) of such systems affect the performance. Furthermore, the effect of sensor and quantisation noise on the overall performance of the system also needs to be considered. This paper describes the development of a mathematical model of a multispectral imaging system that takes into account imaging parameters and noise. The results from the computational model show that increasing the number of colour channels alone in the imaging system does not necessarily allow better estimates of spectral reflectance. The choice of illumination can also, in the presence of noise, greatly affect performance.

Introduction

The colour properties of uniform coloured surfaces can be measured and analysed using the CIE system based upon measurements made using reflectance spectrophotometers. However, as colour measurement and control is being extended from the traditional surface coatings industries to encompass foodstuffs, pharmaceuticals and printed images, there is an increasing need to be able to measure colour properties of complex surfaces or images for which traditional spectrophotometers are not suitable. For example, to measure every distinct colour patch in an image using a spectrophotometer would be at best laborious and at worst impossible because of the small size of the individual coloured regions. There is therefore a growing interest in the development of camera systems that can measure or recover CIE trichromatic coordinates at each pixel location [1,2]. The characterisation of camera systems to enable measurements of *XYZ* values at each pixel location is a step forward but essentially sacrifices colour resolution for spatial resolution. That is, such systems allow *XYZ* values to be quickly captured for each pixel in the image, but only provide a colorimetric measurement and therefore cannot distinguish between spectral metamers. A less compromised solution is to use hyperspectral cameras that are able to measure the spectral colour properties at each pixel location [3]. Such devices are slowly becoming available but are extremely expensive.

Multispectral imaging is a term that is used to describe camera systems that attempt to recover spectral properties using a relatively small number of channels (as few as three). Thus even a standard RGB camera system can be used to estimate the reflectance properties of an image at each pixel location [4]. For example, several researchers have used linear models to recover spectral information from low-cost imaging systems [5–7]. However, research carried out using specific camera systems is often restricted by the limits of the particular system (for example, the number of colour channels) and may not be applicable in a wider sense. Therefore, computational models of imaging systems have been used to efficiently explore the effect of imaging parameters on the ability of the system to recover spectral information [8–9].

In this paper we describe the development of a mathematical model of a multispectral imaging system. We describe experiments that address issues such as how the accuracy of the system varies with the number and spectral properties of the colour channels.

Experimental

Recovering spectral information from camera systems

The response of a camera system to a given point on an illuminated surface is given by Eqn 1:

$$O_i = \int_{\lambda} E(\lambda)R(\lambda)S_i(\lambda)\mathrm{d}\lambda \tag{1}$$

where $S_i(\lambda)$ is the spectral sensitivity of each channel i ($i = 1,\ldots,P$), $E(\lambda)$ and $R(\lambda)$ refer to the spectral power of the illumination and the surface reflectance functions of wavelength λ, respectively, and O_i is the output of the camera's sensor array for each class of sensor i.

If the response of a system with three channels ($P = 3$) is considered at discrete wavelengths, however, we can write a description of the system as follows:

$$\begin{aligned} O_1 &= M_1(\lambda_1)R(\lambda_1) + M_1(\lambda_2)R(\lambda_2) + \ldots + M_1(\lambda_n)R(\lambda_n) \\ O_2 &= M_2(\lambda_1)R(\lambda_1) + M_2(\lambda_2)R(\lambda_2) + \ldots + M_2(\lambda_n)R(\lambda_n) \\ O_3 &= M_3(\lambda_1)R(\lambda_1) + M_3(\lambda_2)R(\lambda_2) + \ldots + M_3(\lambda_n)R(\lambda_n) \end{aligned} \tag{2}$$

where the illuminant and sensor spectral sensitivity terms have been combined to give a single function of wavelength for each channel M_i. If the system is assumed to be represented at each of 31 wavelengths (for example, 400–700 nm at 10 nm intervals) then the response of each channel can be considered to be the sum of 31 terms. It is somewhat convenient to alternatively represent Eqn 2 as a single linear system, as shown in Eqn 3:

$$\mathbf{o} = \mathbf{Mr} \tag{3}$$

where $\mathbf{o}$ is a 3×1 column vector of the camera responses, $\mathbf{M}$ is a 3×31 matrix (whose rows represent the spectral response of the camera multiplied by the spectral distribution of the illuminating source), and $\mathbf{r}$ is a 31×1 column vector of the reflectance spectrum. Eqn 3 is interesting since it implies that the reflectance vector $\mathbf{r}$ can be recovered (Eqn 4):

$$\mathbf{r} = \mathbf{M}^{-1}\mathbf{o} \tag{4}$$

where $\mathbf{M}^{-1}$ is the inverse of the system matrix $\mathbf{M}$.

Unfortunately, if only three sensor responses are available and we wish to recover spectral reflectance at 31 wavelengths, then Eqn 4 is underdetermined and the estimate of $\mathbf{r}$ is likely to be very poor. Thus it would seem that if we want to recover $\mathbf{r}$ at 31 wavelengths then we would need a camera system with 31 channels. Such a system would be a hyperspectral imaging system and would be expensive to manufacture. The interesting question is whether any reliable estimates of $\mathbf{r}$ can be achieved with a low-cost RGB camera system.

One possible answer to this question is based upon the fact that the vast majority of reflectance spectra for natural and man-made surfaces are smooth functions of wavelength [10]. The smoothness can be incorporated by a linear model in which each reflectance spectrum $R(\lambda)$ is represented as the weighted sum of a small number N of basis functions $B_k(\lambda)$ (Eqn 5):

$$R(\lambda) \approx \sum_k a_k B_k(\lambda) \tag{5}$$

where a_k are weighting coefficients and $k = 1,\ldots,N$.

If these basis functions are chosen carefully, as few as six such functions can be used to closely approximate most man-made and natural reflectance spectra. The term for reflectance in Eqn 1 can be replaced by the approximation given in Eqn 5 to give Eqn 6:

$$O_i = \int_\lambda E(\lambda)\left(\sum_k a_k B_k(\lambda)\right) S_i(\lambda)\mathrm{d}\lambda \tag{6}$$

If we assume that $E(\lambda)$, $S(\lambda)$ and $B_k(\lambda)$ are all known, Eqn 6 can be recast as before as a matrix-algebra problem with P equations and N unknowns. This gives Eqn 7:

$$\mathbf{o} = \mathbf{La} \tag{7}$$

where $\mathbf{a}$ is a column vector of coefficients, $\mathbf{o}$ is now a $P \times 1$ column vector of digital output values and $\mathbf{L}$ is a $P \times N$ matrix obtained by the product of the illuminant, spectral sensitivity curves of the camera and the basis functions. Eqn 7 can be solved to yield $\mathbf{a}$ by determining the inverse of $\mathbf{L}$, denoted by $\mathbf{L}^{-1}$, and multiplying through to give:

$$\mathbf{L}^{-1}\mathbf{o} = \mathbf{a} \tag{8}$$

Once $\mathbf{a}$ is known the elements of this vector give the weighting coefficients a_k directly and the estimated reflectance is then calculated using Eqn 5.

The assumption that $E(\lambda)$, $S(\lambda)$ and $B_k(\lambda)$ are all known is a reasonable one since $E(\lambda)$ and $S(\lambda)$ can be measured or estimated for a controlled viewing environment and a specific camera system and the basis functions $B_k(\lambda)$ can be computed from a suitable set of reflectance spectra.

Methodology

This study employed a camera model that makes several assumptions. Firstly, channel sensitivities $S_i(\lambda)$ were always assumed to be Gaussian functions of wavelength. Secondly, the maximum sensitivities of the channels were placed at equal intervals along the visible spectrum. Thirdly, each sensor was normalised such that the integral under the sensitivity curve had a value of 1. Fourthly, the number of basis functions N used in the recovery process was always equal to the number of sensor channels P so that the solution matrix $\mathbf{L}$ (see Eqn 7) was always square. These assumptions were made to constrain the camera properties so that they could be investigated systematically. Thus, the channel sensitivities were represented by the number of channels and the broadness of their spectral envelopes. Of course, these assumptions tended to simplify the model in comparison with a real camera system but nevertheless they are not unrealistic. The spectral sensitivities of many commercial cameras, for example, are to a first-order approximation Gaussian functions of wavelength [11]. The most serious assumption was probably that of matching the number of basis functions in the linear model of reflectance with the number of channels. The effect of relaxing this constraint will be discussed in a subsequent publication.

The values of $E(\lambda)$ tested represented CIE illuminant A, F11, D65 and an equal-energy illuminant with a value of 1 at all wavelengths. The imaging parameters investigated were the number of sensors P and the sensor half width σ. The effect of changing these parameters was evaluated in the presence of random noise, quantisation noise and without noise. In the case of random noise, small random values (normally distributed with mean zero and a variable standard deviation, SD) were added to the sensor outputs. In the case of quantisation noise the sensor outputs were rounded to simulate their analogue-to-digital conversion into discrete 8-, 10- and 12-bit representations. Thus the camera outputs of our model were initially represented by any value in the range 0–1. In order to model 12-bit digital resolution, for example, these continuous values were scaled to the range 0–4095 and then rounded to the nearest integer.

A set of measured reflectance spectra (400–700 nm at 10 nm intervals) representing 1269 Munsell chips was obtained [12] and used in this study. Thus, for various values of P and σ, the output of the camera model O_i was computed using Eqn 1 for each of 1269 samples using various noise and illuminant conditions. The sensor outputs for each sample were then used to recover estimates of the reflectance functions of that sample using the recovery process outlined earlier. The performance of the model was tested by comparing the reconstructed spectra with the original spectra and representing the differences as CIELab ΔE values (reconstruction errors) under illuminant D65. The model was implemented using the MATLAB programming environment.

A standard method for computing the basis functions for a set of reflectance spectra employs singular value decomposition and is provided as a single command SVDS in MATLAB. The basis functions used (see Eqn 5) were derived using this method and based upon the 1269 samples in the Munsell set.

Results and Discussion

Effect of sensor number

Figure 1 shows the median ΔE reconstruction error as a function of sensor number (σ = 110 nm) for various illuminants for the no-noise condition. Median values are more appropriate than mean values because the ΔE values are almost certainly not normally distributed [13].

It is evident that, in general, performance improves (ΔE decreases) with increasing number of sensors. Recall that the number of basis functions used to represent the reflectance functions increases with the number of sensors (thus, $P = N$). However, performance did not always improve with increasing the number of sensors. For example, pronounced 'peaks' in median ΔE are clearly evident when $P = 6$ for all illuminants.

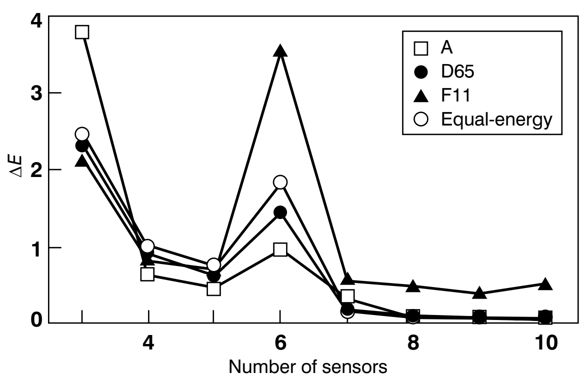

Figure 1 Number of sensors for four illuminants vs. median ΔE reconstruction error (σ = 110 nm)

Effect of quantisation noise

Figure 2 shows the effect of increasing quantisation noise on the reconstruction error for different numbers of sensors (data are shown for σ = 110 nm and the equal-energy illuminant condition). As expected, the effect of increasing quantisation noise was to increase the error but the effect was more marked for high values of P. The effect was also more marked with greater values of σ.

Effect of random sensor noise

Figure 3 shows the result of increasing the standard deviation, SD, of the random noise and, unsurprisingly, the reconstruction error increases with increasing SD. We also investigated the relationship between the effects of random

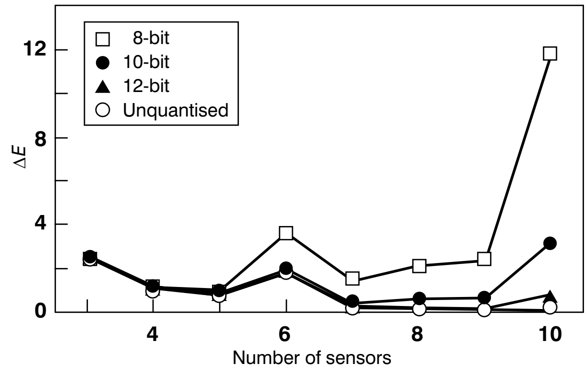

Figure 2 Number of sensors for four levels of quantisation noise vs. median ΔE reconstruction error (σ = 110 nm; equal-energy illuminant)

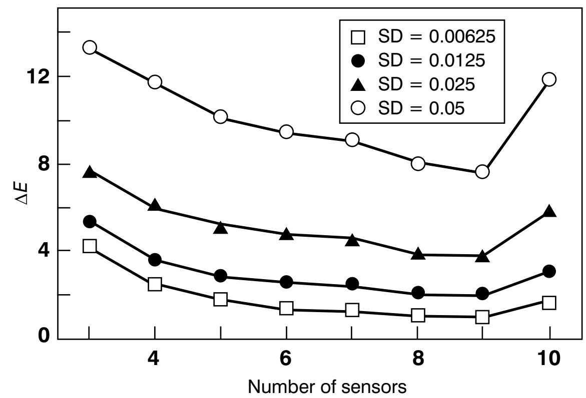

Figure 3 Number of sensors for four different levels of standard deviation vs. median ΔE reconstruction error (σ = 110 nm; equal-energy illuminant)

noise and sensor half width and found that the effect of noise on the reconstruction error was more marked for larger values of σ.

Effect of illuminant

In the no-noise condition there was no consistent difference in results obtained using different illuminants. However, Figure 4 compares the results for the different illuminants in the presence of random noise. High errors can be

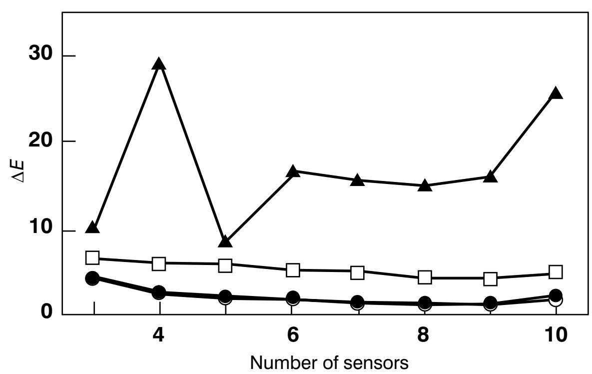

Figure 4 Number of sensors for four illuminants in the presence of random noise vs. median ΔE reconstruction error (σ = 110 nm; SD = 0.00625); for key see Figure 1

observed for illuminants A and F11. Conversely, illuminants D65 and the equal energy illuminant appear to make the reconstruction process relatively robust to sensor noise.

Analysis of the reconstruction error results

The results of this study demonstrate several principles. Firstly, it is not necessarily advantageous to increase the number of sensors. Reasonable performance is obtained with $P = 5$ (median $\Delta E < 1$) but the error can increase dramatically when $P = 6$ (median $\Delta E > 4$ for some illuminants) (Figure 1). This confirms findings made independently by Hernández-Andrés *et al*. who found that when illumination spectra were estimated from small numbers of channels the reconstruction error did not consistently fall with increasing numbers of channels [9]. Secondly, although reconstruction errors are relatively independent of the choice of illuminant in noise-free conditions, in noisy conditions there is a clear advantage for using illuminants which have an equal distribution of energy across all wavelengths. It is also true that in the presence of quantisation noise, increasing the number of sensors beyond 5 is detrimental to performance (Figure 2). In related experiments we have also found that the reconstruction error decreases with the half-width of the sensor under noisy conditions [14].

In fact, we have found that the circumstances under which the reconstruction error is high closely correspond with the linear system (Eqn 7) being ill-conditioned. This property of the linear system can be quantified by the condition number [15] of the matrix **L** (Eqn 7) which can be computed as the product of the norms of the matrix and its inverse thus $||\mathbf{L}|| \times ||\mathbf{L}^{-1}||$. We find that when the condition number of **L** is high the process is more sensitive to noise. Furthermore, condition number generally increases with increasing P and σ, and is much higher for illuminants with uneven energy distributions. Thus, the rather surprising result that increasing the number of colour channels in the system does not necessarily reduce the reconstruction error may be explained in terms of the condition number of the system matrix. In order to minimise error in the reconstruction process it may be necessary to choose sensors and illuminants such that they minimise the condition number of **L**.

When the condition number is high this implies that coefficients in the columns of the matrix **L** are becoming increasingly correlated. Changing the wavelength spacing of the peaks of the channels would almost certainly be a major factor that determines the condition number. It is important to note, therefore, that the argument proposed is not that having six sensors in any camera system will always result in relatively poor performance – since relaxing the constraint of equally spaced sensors may well have produced different results – but that increasing the number of sensors alone does not guarantee better performance. The interaction of all of the camera properties (in our case, number of channels, width of channel sensitivities, etc) needs to be considered.

Conclusions

It is important to note that our camera model included several assumptions that may not all be met by a real imaging system. Furthermore, there are a number of potential parameters that we have not yet investigated such as the wavelength spacing of the channel peaks and different mathematical methods for estimating reflectance. However, a number of design criteria seem to have emerged from the present study. For example, best results are generally achieved when illuminants with flat spectral power distributions are used, the half-width of the channel spectral properties is narrow, and the number of channels is relatively few if the channel half-widths are large.

The next stage of the study is to verify the correctness of the model by comparing the predictions made by the model with data obtained from a real multispectral imaging system that is being constructed at Derby University. The purpose of such a comparison is to confirm that a linear model (Eqn 1) for the camera system is valid and to provide realistic estimates for noise parameters in this model. The final stage of the project will be the development of an optimisation procedure to allow the parameter space of the system to be searched and the set of parameters that allows the lowest reconstruction error to be determined. Such a procedure would provide an efficient design tool for effective multispectral imaging systems based upon low-cost camera systems.

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