

A Study of Metameric Blacks for the Representation of Spectral Images

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Abstract. Spectral images contain a large volume of data and can be efficiently compressed by low dimensional linear models. However, there is a trade-off between the accuracy of spectral and colorimetric representation. When a spectral image is reproduced by a low-dimensional linear model, spectral error and color difference are contrary to each other and minimizing the colour error is by no means equivalent to minimizing the spectral error. Although one aim of a spectral-image file format is to preserve and represent the spectral information, most users are likely to reproduce a spectral image on a trichromatic image-reproduction system and therefore it is important that the spectral information is not preserved at the expense of colorimetric accuracy. In this study a method for spectral encoding that provides an efficient representation of the spectral information whilst perfectly preserving the colorimetric information is analysed. The lossy compression technique that is considered in this work is based on a low-dimensional linear model of spectral reflectance, with the first three basis functions represent color information and the additional basis functions are metameric blacks which preserve spectral information.

Introduction

Spectral images contain a huge volume of data and the development of multispectral imaging systems places considerable demands on computer hardware and software compared with standard three-component or trichromatic image storage and processing. However, a spectral image containing very huge spectral information and it is highly desirable to find a way to represent spectral images efficiently by compressing them into a more compact form. The lossy compression techniques for spectral images are applicable since many color images are intended for display for human perception and it is well established that images contain redundancies (in terms of their color, spatial and temporal properties) that can be removed without any loss in image quality.

The lossy compression technique that is considered in this work is a linear model of spectral reflectance. Any spectral reflectance distribution $P(\lambda)$ can be approximated to a specified degree of accuracy as a weighted sum of basis functions $B(\lambda)$, thus

$$P(\lambda) = a_1 B_1(\lambda) + a_2 B_2(\lambda) + \dots + a_n B_n(\lambda) . \quad (1)$$

where $P(\lambda)$ is the reflectance spectrum, $B_i(\lambda)$ is the i th basis function and a_i is the coefficient or weight for the i th basis function [1]. Once the $B_i(\lambda)$ are known, a set of weights a_i is sufficient to specify any reflectance spectrum in the model. It has been stated that spectral reflectance of surfaces and spectral power distributions of illuminants are highly constrained [2,3,4], therefore they can be approximated by low-dimensional linear models of limited weighted sum of basis functions.

In the metameric black theory [5], a spectral reflectance $p_b(\lambda)$ is generated which for a given illuminant $E(\lambda)$ and a given observer $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$ fulfils the conditions describe in Equation 2

$$\begin{aligned}
 \sum_{\lambda} p_b(\lambda)E(\lambda)\bar{x}(\lambda)\Delta\lambda &= 0 \\
 \sum_{\lambda} p_b(\lambda)E(\lambda)\bar{y}(\lambda)\Delta\lambda &= 0 \\
 \sum_{\lambda} p_b(\lambda)E(\lambda)\bar{z}(\lambda)\Delta\lambda &= 0
 \end{aligned} \tag{2}$$

Thus the tristimulus values calculated for a metamer black are zero. All metamer blacks have the important feature of being positive at some wavelength and negative at others, and thus, they do not physically exist (see example in Figure 1). But they can be used to construct sets of all positive spectral reflectance functions that are metameric and match some colours other than black. The procedure is to add a spectral reflectance function $p_b(\lambda)$ of a metamer black to a given spectral reflectance function $p_0(\lambda)$ of non-vanishing tristimulus values to obtain a new spectral reflectance function $p(\lambda)$:

$$p(\lambda) = p_0(\lambda) + ap_b(\lambda) \tag{3}$$

where a is a scaling factor.

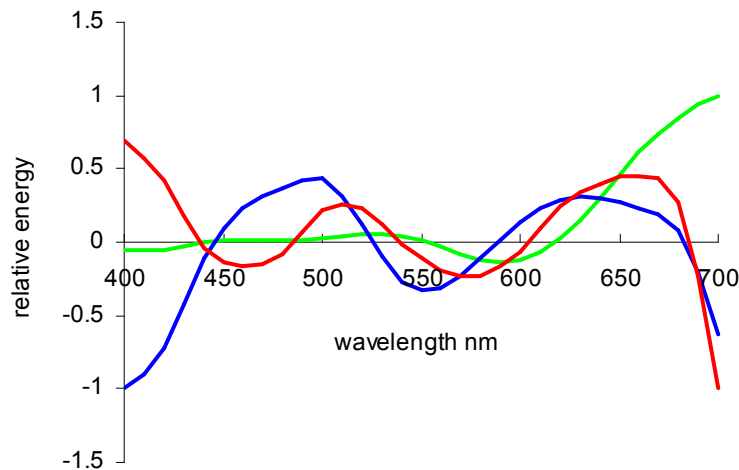


Figure 1: An example of a set of metamer blacks.

The metamer black has been proved to be useful in applications of colour correction [6], camera calibration [7] and spectral image encoding [8]. Our approach is based on the idea of spectral image encoding proposed by Keusen and Praefcke [8] and a technique mentioned by Fairman and Brill [9]. The first three basis functions used are derived from reflectance spectra data by the conventional linear modelling (zero-centred PCA) and the additional basis functions are metamer blacks which do not disrupt the colorimetric properties of the first three basis functions.

Experimental

In this study, the conventional linear model which is a weighted sum of basis functions is modified to represent spectra. The first three basis functions as such derived from conventional linear modelling while metamer blacks calculated to do a spectral match.

Consider that a given reflectance vector \mathbf{p} will have tristimulus values \mathbf{t} for a defined illuminant and a set of CIE colour-matching functions. The reflectance vector \mathbf{p} is represented by a linear model such that the tristimulus values \mathbf{t} is unchanged. Thus,

$$\mathbf{p} \approx \mathbf{B}\mathbf{w} \quad (4)$$

where \mathbf{B} is a $m \times n$ matrix of the n basis functions, \mathbf{w} is the $n \times 1$ vector of weights, and \mathbf{p} is the $m \times 1$ reflectance vector represented at the m wavelength intervals. The first three basis functions \mathbf{B}_{1-3} were selected for the reproduction of tristimulus values. The first three basis functions \mathbf{B}_{1-3} were derived by a singular-value decomposition of a set of reflectance spectra which is the zero-centred principal component analysis (PCA). The weights for these first three basis functions are computed by a colorimetric match to the tristimulus vector \mathbf{t} . Thus,

$$\mathbf{t} = \mathbf{M}\mathbf{p} \quad (5)$$

where \mathbf{M} is a $3 \times m$ matrix whose rows are the colour-matching functions multiplied by the illuminant, and now by representing \mathbf{p} by the first three basis functions we can write

$$\mathbf{t} = \mathbf{M}(\mathbf{B}_{1-3}\mathbf{w}_{1-3}) \quad (6)$$

The first three basis weights \mathbf{w}_{1-3} can now be calculated by grouping together the \mathbf{M} and \mathbf{B}_{1-3} matrices to produced a 3×3 square matrix \mathbf{Q} thus

$$\mathbf{w}_{1-3} = \mathbf{Q}^{-1}\mathbf{t} \quad (7)$$

where \mathbf{Q}^{-1} is the inverse of \mathbf{Q} . Thus one reflectance spectra can be reconstructed by:

$$\mathbf{p}_{1-3} = \mathbf{B}_{1-3}\mathbf{w}_{1-3} \quad (8)$$

The reconstructed reflectance \mathbf{p}_{1-3} reproduced is now a metamer to \mathbf{p} ; that is both \mathbf{p}_{1-3} and \mathbf{p} have the same tristimulus values. And a set of reflectance data can be reconstructed by:

$$\mathbf{P}_{1-3} = \mathbf{B}_{1-3}\mathbf{W}_{1-3} \quad (9)$$

where \mathbf{P}_{1-3} is the $m \times k$ reflectance matrix representing k samples at the m wavelength intervals, and \mathbf{W}_{1-3} is the $3 \times k$ matrix of basis weights. At this stage a linear model representation is obtained as \mathbf{P}_{1-3} that preserves the colorimetric information about the original reflectance data set \mathbf{P} under a specified illuminant.

Although the use of the first three basis functions and the computation of basis weights via a colorimetric match preserve the colorimetric information well, the accuracy of the spectral information may not be optimized. One way to preserve the spectral information well is to modify the spectrum without changing its tristimulus value and the idea of spectral match by adding metameric blacks has been carefully considered. In this study a set of metameric black basis functions is calculated with the zero-centred PCA technique based on the given three basis function and the reflectance data set \mathbf{P} for encoding.

$$\mathbf{P}_s = \mathbf{P} - \mathbf{P}_{1-3} \quad (10)$$

where \mathbf{P}_s is the modified data set spanning a vector subspace. Basis functions matrix \mathbf{B}_{4-n} derived from \mathbf{P}_s which contains k columns of metameric blacks were therefore used, and each basis functions \mathbf{B}_i ($4 \leq i \leq n$) is supposed to satisfy the following constraint:

$$\mathbf{T}_0 = \mathbf{M} \mathbf{B}_i \quad (11)$$

where \mathbf{T}_0 is a $3 \times n$ matrix of zeros tristimulus values.

So the basis encoding method for the representing spectral images can be described by

$$\mathbf{P} = \mathbf{B}_{1-3} \mathbf{W}_{1-3} + \mathbf{B}_{4-n} \mathbf{W}_{4-n} \quad (12)$$

where \mathbf{B}_{4-n} are metameric blacks derived from the original basis functions. The basis weights \mathbf{W}_{4-n} are computed for the metameric blacks by making a spectral match to the residues of the original reflectance spectrum \mathbf{P} minus the metamer $\mathbf{B}_{1-3} \mathbf{W}_{1-3}$.

As the reconstructed reflectance and the original reflectance are supposed to be metamerism under illuminant that determine \mathbf{M} , the colour difference under such illuminant will be zero, so metamerism index were used as the colorimetric metrics to judge reconstruction errors with such illuminant as reference and other two as tests. The reference illuminant is chosen as the primary illuminant from which the basis functions are derived, so the metamerism index for other illuminant is the same as CIELAB colour differences under the test illuminant. In this experiment, three illuminants D65, A and F11 were selected as primary illuminants, for each primary illuminant, a linear models derived from 1269 Munsell reflectance spectra by the approach described above was used to approximate the original data set (3 to 9 basis functions were used), CIELAB ΔE^*_{ab} with test illuminants of D65, A and F11 (with CIE 1931 standard observer) and RMS were calculated.

Results

The linear models were derived from 1269 Munsell reflectance data set. The illuminant was selected as D65 and standard observers were selected as CIE 1931 colour matching functions for example, and 9 basis functions in total were computed. Figure 2 illustrates the spectral curves of the corresponding sets of basis functions derived from this method.

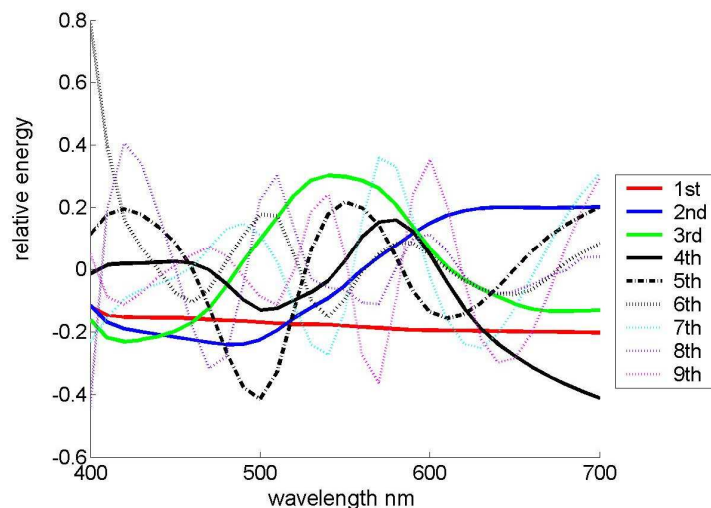


Figure 2 : The first 9 basis functions derived from 1269 Munsell reflectance samples by Method 2.

In general, with the increased number of blacks, the curve of corresponding metameric black shows more peaks and thus presents more high frequency information of the original spectral reflectance. By the way we derived metameric blacks which are the subsequent basis functions, it is not guaranteed that each basis functions are orthogonal with each other. If two vectors of basis function \mathbf{B}_i and \mathbf{B}_j are orthogonal with each other, they should satisfy that:

$$\mathbf{B}_i^T \mathbf{B}_j = 0 \quad (13)$$

where \mathbf{T} is the matrix transpose operator. So for each set of basis functions, the orthogonality between each i th and j th basis function were tested. It has been found that for Method 1, each basis function is orthogonal with each other, while for Method 2 and Method 3, each of the first 3 basis functions is orthogonal with each other and each metameric black is orthogonal with each other, but each of the first three basis functions is not orthogonal with each metameric black.

Reference illuminant	Number of basis functions	Test illuminant								
		ΔE^*_{ab} (D65)		ΔE^*_{ab} (A)		ΔE^*_{ab} (F11)		RMS (%)		
		mean	max	mean	max	mean	max	mean	max	
D65	3	0.00	0.00	1.63	10.85	1.98	11.92	2.34	15.36	
	4	0.00	0.00	0.92	13.39	1.91	13.43	1.43	7.00	
	5	0.00	0.00	0.26	4.81	1.10	14.70	1.01	5.42	
	6	0.00	0.00	0.26	4.76	1.02	10.19	0.76	5.17	
	7	0.00	0.00	0.07	0.79	0.76	10.29	0.54	2.99	
	8	0.00	0.00	0.04	0.72	0.65	11.54	0.41	2.94	
	9	0.00	0.00	0.03	0.33	0.48	4.30	0.29	1.34	
	A	3	1.88	17.05	0.00	0.00	1.87	17.22	2.18	13.07
		4	1.26	18.79	0.00	0.00	1.52	13.45	1.50	7.08
5		0.30	5.22	0.00	0.00	1.18	19.84	1.02	5.29	
6		0.27	5.67	0.00	0.00	1.17	16.61	0.76	5.14	
7		0.07	0.79	0.00	0.00	0.80	10.78	0.54	3.00	
8		0.04	0.42	0.00	0.00	0.65	11.97	0.41	2.98	
9		0.04	0.37	0.00	0.00	0.49	3.96	0.29	1.34	
F11		3	1.93	11.05	1.74	14.26	0.00	0.00	2.41	15.37
		4	1.90	11.52	1.49	13.98	0.00	0.00	1.65	7.73
	5	0.97	15.70	0.95	17.86	0.00	0.00	1.05	5.64	
	6	0.61	7.04	0.59	7.16	0.00	0.00	0.85	3.66	
	7	0.57	6.70	0.55	7.52	0.00	0.00	0.56	3.60	
	8	0.52	6.82	0.49	6.33	0.00	0.00	0.45	2.01	
	9	0.31	3.11	0.29	2.61	0.00	0.00	0.31	1.39	

Table 1: Reconstruction errors under D65, A and F11

For the full set of Munsell samples, the reconstruction errors in CIELAB ΔE^*_{ab} and RMS are shown in Table 1. It can be seen that all the results follow the rules that the more basis functions the smaller the reconstruction errors, which are compatible with all the previous results of using conventional linear models, and the trends are consistent with spectral RMS errors and colour differences under all three illuminants. It is noticed from both RMS and colour differences that when the 4th metameric blacks are introduced, there is a significant drop. When it comes to the variations of illuminants under which the tristimulus values are preserved, it can be seen that the spectral RMS errors under each illuminant are very similar. Achieving tristimulus match under F11 will always produce largest spectral reconstruction errors while much more similar results are shown under D65 and A. General speaking, the method gives a good performance by always producing small spectral errors and small colour differences to matching tristimulus values under each of 3 illuminants. According to

Table , if using CIELAB ΔE^*_{ab} of 1 unit as a criteria to judge the number of basis functions required for the 1269 Munsell data set, to match tristimulus values under D65 and A, 6 basis functions are required under test illuminant A and D65 respectively and one more for the test

illuminant of F11; while to match tristimulus values under F11, 8 basis functions are required for both D65 and A as test illuminants.

Conclusions

In this study, a way of spectral reflectance representation has been investigated to preserve the tristimulus values under certain primary illuminant and observers, and fine-tune the spectral curve without changing its colorimetric match. Metameric black technique was therefore introduced to satisfy this condition. The proposed approach is taking the first three basis functions are the basis functions derived from the data set its own. While the subsequent metameric blacks were derived by two ways, one is the basis functions of residues, another is the transformations of basis functions derived from the own data set. Results of analysis on Munsell reflectance samples show that the first three basis functions is orthogonal with each other and each of the metameric blacks is orthogonal with each other, but each of the first three basis functions are not orthogonal with each of the metameric blacks.

In general, the conventional linear modelling presents smallest spectral errors, while not necessarily the smallest colorimetric errors. While linear modelling based on metameric black technique presents perfect tristimulus matching under the primary illuminant. But there is always a contradiction between minimizing spectral error and colorimetric error, conventional linear model gives the best spectral match but worst colorimetric match, linear model based on metameric black technique gives the best colour match but can not be the best spectral match.

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