Model of Luminance Contrast-Sensitivity Function for Application to Image Assessment

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Abstract: The contrast-sensitivity function (CSF) is a measure of fundamental spatiochromatic properties of the human visual system. It is typically measured at the detection threshold for the psychophysically defined cardinal channels: luminance, red–green, and yellow–blue. Recent measurements of luminance contrast sensitivity show that the sensitivity of the luminance channel is less for chromatic stimuli than for achromatic stimuli. This chromatic effect has important implications for both human and machine vision. The chromatic effect on luminance contrast sensitivity has been modeled in this work based upon an existing model published by Barten. Barten’s model was chosen as the starting point for this work because it is analytical, relatively simple, and can predict those effects that are important for image analysis.

INTRODUCTION

The standardization of the colour-response properties of the human visual system, in terms of the Commission Internationale de l’Eclairage (CIE) colour-matching functions, has allowed great advances to be made in colorimetry that have impacted the development of new technologies in a wide range of industries. The imaging community has particularly benefited from the CIE standards and recommendations. However, there currently exists no equivalent standard for the spatiochromatic properties of the visual system. Such a standard would most naturally be expressed in terms of the contrast-sensitivity function (CSF). The CSF is a measure of fundamental spatiochromatic properties of the human visual system. It is typically measured at the detection threshold for the psychophysically defined cardinal channels: luminance, red–green, and yellow–blue. Thus, for the luminance channel, the detection thresholds for chromatically neutral stimuli—sinusoidal gratings of a certain spatial frequency—are measured and the sensitivity is expressed as an inverse of the detection threshold. Numerous psychophysical measurements of the luminance CSF have demonstrated that it has band-pass shape; that is, sensitivity to luminance contrast peaks at about 6 cycles/degree and falls off at higher and lower spatial frequencies.¹–³ The fall-off in sensitivity at higher spatial frequencies can be attributed to the eye’s optics and to spatial limitations in the retinal mosaic of cone receptors. The reduction in sensitivity at lower spatial frequencies requires neural explanations and therefore the psychophysically measured CSF is not simply the inverse of the modulation transfer function of the eye.

There is a great deal of evidence to support the notion that the visual system is adaptive.⁴ In the case of CSF this means that the magnitude and shape of the CSF change with certain properties of the stimulus and this has made standardization of the functions difficult. So, for example, the luminance CSF generally decreases with mean luminance and this is accompanied by a shift in peak sensitivity to lower spatial frequencies (indeed, for stimuli of very low mean luminance the luminance CSF becomes low pass).⁵ Note that if Weber’s law was valid the luminance CSF would remain constant for stimuli having different mean luminance. The reduction in luminance contrast sensitivity that is observed when the luminance of the stimulus is reduced therefore represents a breakdown in Weber’s law.⁶ Other measurements have revealed that sensitivity to luminance contrast reduces (and become more low pass) with increased stimulus eccentricity⁷ and with decreased stimu-
Various models of luminance CSF have been published and are widely applied in imaging analysis. For example, Barten has developed two models: one that is relatively complex and physiologically inspired and another that is simpler and empirically fitted to psychophysical data. The latter model is reproduced as Eq. (1),

$$\text{CSF} = a f e^{-b f} (1 + c e^{b f})^{0.5},$$  

where

$$a = \frac{[540(1+0.7/L)^{-0.2}]/[1+12(1+f/3)^{-2}/w]},$$

$$b = 0.3(1+100/L)^{0.15},$$

$$c = 0.06,$$

and where $f$ is the spatial frequency of the stimulus, $w$ is the stimulus size in degrees of visual angle, and $L$ is the mean luminance of the stimulus in cd/m$^2$. Figure 1 shows the model predictions for stimuli with various mean luminance values and a fixed size (10°). Estimates of the luminance CSF (and also the equivalent functions for the two chromatic channels) are frequently employed in computational models that attempt to predict image quality or the perceptibility of differences between a pair of images. The S-CIELAB model, for example, employs such estimates to predict the visible difference between an original image and its reproduction.

Two male observers (24 and 31 years old) with normal colour vision participated in the experiments whereby full-field (8 × 11°) vertically oriented luminance-modulated stimuli were presented on a CRT at distance of approximately 1 m in a darkened room. Observers were instructed to fixate to a central point and to alter the luminance contrast for stimuli until the grating was just visible. Under the standard condition the stimuli were achromatic with a mean luminance of 30 cd/m$^2$. Similar experiments were also carried out at the same mean luminance level but with chromatic backgrounds. The detection thresholds were measured for each observer at each spatial frequency for achromatic and chromatic backgrounds.

This finding (Fig. 3) has important implications for the use of the CSF in image-analysis models. The use of a luminance CSF measured for a chromatically neutral stimulus and then used for the luminance component of a colour image implies...
that the sensitivity of the visual system to luminance is independent of the strength of any chromatic signals. However, if sensitivity to luminance contrast is reduced in the presence of a chromatic signal then such use effectively overestimates the visual system’s sensitivity to luminance contrast in colour images. What is required for application in imaging models is a model of luminance CSF that incorporates chromatic dependence. This article is concerned with the development of such a model of luminance CSF.

**EXPERIMENTAL PROCEDURES**

The model developed by Barten (Eq. (1)) has been taken as the starting point for this work. This model was chosen because it has been used by numerous researchers in various imaging experiments, is of a relatively simple analytic form, and is able to predict important properties of the CSF that have been published (such as the change in CSF with the mean luminance and the size of the stimulus). There are other models that are able to predict the effect of stimulus eccentricity but this effect is of no consequence when the CSF is used as a spatially global filter in an imaging application such as S-CIELAB. The generic form of the model that has been used is expressed as Eq. (2),

$$\text{CSF}(f) = a e^{-bf(1+ce^{bf})^{0.5}},$$  

(2)

where

$$a = [1000p(1)(1+0.7/L)^{-0.2}][1+12(1+f/3)^{-2}/w],$$

$$b = p(2)(1+100/L)^{0.15},$$

$$c = p(3),$$

and where $p(i) \in \{1,2,3\}$ are variable coefficients that will enable the model to fit the psychophysical data. Modeling fitting was performed using MATLAB’s `fminsearch` function, which performs a multidimensional unconstrained nonlinear minimization (Nelder–Mead). The Nelder–Mead method is a simplex method for finding a local minimum of a function of several variables, in this case $p(i)$. The cost function that was minimized was the root-mean-squared error between the psychophysical data and the predictions of contrast sensitivity by the model (Eq. (2)). The starting values for all cases were that $p(i) = 0.5, i \in \{1,2,3\}$. The psychophysical data that were fitted are given in Table I.

**RESULTS**

The model described by Eq. (1) was fitted to the psychophysical data (Table I). For the luminance CSF data obtained using the achromatic background condition the values obtained were $p(1) = 0.6349$, $p(2) = 0.2186$, and $p(3) = 0.1434$ and the Pearson product moment correlation coefficient $r^2 = 0.9906$. For the luminance CSF data obtained using the chromatic background condition the values obtained were $p(1) = 0.1570$, $p(2) = 0.2413$, and $p(3) = 0.5287$ and the Pearson product moment correlation coefficient was 0.9988. A visual indication of the quality of fit for these two situations is given in Fig. 4.

It is evident that the generalized form of Barten’s model is able to fit our psychophysical data for both the achromatic- and chromatic-background conditions. It would be desirable, however, to have a computational model of luminance CSF

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**TABLE I. Psychophysical data for luminance csf for the achromatic- and chromatic-background conditions.**

<table>
<thead>
<tr>
<th>Spatial frequency (cycles/degree)</th>
<th>2.53</th>
<th>5.29</th>
<th>9.70</th>
<th>19.39</th>
<th>29.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>Achromatic</td>
<td>702.50</td>
<td>831.00</td>
<td>688.00</td>
<td>368.00</td>
<td>139.01</td>
</tr>
<tr>
<td>Chromatic</td>
<td>199.00</td>
<td>270.75</td>
<td>252.25</td>
<td>120.19</td>
<td>36.71</td>
</tr>
</tbody>
</table>

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that is able to predict the CSF for a variety of chromatic-background conditions. We assume that the reduction in sensitivity of the luminance channel in the presence of a chromatic signal is a simple scaling relationship. We further assume that the scaling term is linearly related to the chromaticness of the image or stimulus. We therefore refit the model to the psychophysical data for the chromatic-background condition with the values for $p(2)$ and $p(3)$ fixed to those values that were found for the achromatic-background condition.

The dashed line in Fig. 5 illustrates the fit of the scaled achromatic-background model to the chromatic-background data. The goodness of the fit indicates that the scaling assumption is reasonable and the summary in Table II shows that the introduction of this assumption only resulted in a reduction in the correlation coefficient from 0.9988 to 0.9724.

The reduction in sensitivity to contrast for the luminance channel in the presence of a chromatic signal seems to be robust to chromatic backgrounds other than cyan and blue. However, further experiments are urgently required to ascertain the precise relationship between the reduction in sensitivity and the chromaticness of the stimulus. For the present, we assume that the reduction in sensitivity is linearly related to the mean chromaticness of the stimulus computed in an approximately visually uniform colour space such as $u'v'$, where $u'v'$ can be computed from the chromaticity coordinates $xy$ thus,

$$u' = 4x/(-2x + 12y + 3),$$

$$v' = 9x/(-2x + 12y + 3).$$

Figure 6 illustrates the chromaticities of the blue and cyan backgrounds that were used in the psychophysical experiments. We assume that sensitivity falls off as the stimulus moves from the neutral point to the chromatic point in the $u'v'$ diagram. The average distance between the cyan and chromatic backgrounds and the neutral point in the $u'v'$ diagram (Fig. 6) is 0.0642.

The final model of luminance contrast sensitivity can be expressed as Eq. (2) where $p(1) = 0.6349(1 - 10.5102d)$, $p(2) = 0.2186$, and $p(3) = 0.1434$, and where $d$ is the Euclidean distance between the white point and the chromatic background in the approximately uniform $u'v'$ diagram. Figure 7 illustrates the fit of the final model to the psychophysical data. Finally, Fig. 8 illustrates the effect of chromatic background and mean luminance on output of the model, which indicates that the features of Barten’s original model (e.g., reduction of luminance contrast sensitivity with decreasing mean luminance) have been retained but augmented by the new chromatic effect.

**CONCLUSIONS**

Recent measurements of luminance contrast sensitivity show that the sensitivity of the luminance channel is less for chromatic

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**TABLE II. Summary of optimization experiments.**

<table>
<thead>
<tr>
<th>$p(1)$</th>
<th>$p(2)$</th>
<th>$p(3)$</th>
<th>Psychophysical data</th>
<th>$r^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6349</td>
<td>0.2186</td>
<td>0.1434</td>
<td>Achromatic background</td>
<td>0.9906</td>
</tr>
<tr>
<td>0.1570</td>
<td>0.2413</td>
<td>0.5287</td>
<td>Chromatic background</td>
<td>0.9988</td>
</tr>
<tr>
<td>0.2065</td>
<td>0.2186</td>
<td>0.1434</td>
<td>Chromatic background</td>
<td>0.9724</td>
</tr>
</tbody>
</table>

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stimuli than for achromatic stimuli. This chromatic effect has important implications for both human and machine vision. For example, the finding could indicate a lack of independence between the luminance and chromatic channels of the visual processing system. There is some evidence in the literature to support such independence in related tasks.\textsuperscript{14–17} Stromeyer \textit{et al.}, for example, showed that there is chromatic suppression of cone inputs to the luminance flicker mechanism\textsuperscript{14}; intense red fields selectively suppressed the \textit{L} cone input to the luminance mechanism by a factor greater than Weber’s law. Earlier evidence that intense red and green fields suppress the \textit{M} and \textit{L} cone inputs to the luminance mechanism had been provided by Eisner and MacLeod.\textsuperscript{15} Recently, Gur and Akri reported that when luminance and colour contrast are both modulated the luminance CSF is enhanced compared with when it is measured in an achromatic stimulus.\textsuperscript{16} They argued that the use of isoluminant stimuli to study the chromatic channels is only justified if the implicit assumption that the luminance and chromatic channels are independent is true. Knoblauch \textit{et al.} have also shown that the sensitivity to luminance contrast is increased in the presence of chromatic contrast.\textsuperscript{17} Owens’ psychophysical results also fail to support the notion that luminance and chromatic information are processed independently but are somewhat contrary to those of Gur and Akri\textsuperscript{16} and Knoblauch \textit{et al.}\textsuperscript{17} There is thus mounting evidence that luminance and colour information may not be processed independently and this is supported by some other related studies.\textsuperscript{18,19}

There is a strong interest in the imaging community in metrics for the assessment of image quality and for the quantification of differences between images, so-called image-difference metrics. The CSFs of the visual system are central to many such metrics.\textsuperscript{9,10} The reason for this is that the interest is normally not in the physical differences between two images (in the case of image-difference metrics) but in the perceptual differences. The CSF can be used to “throw-away” differences that cannot be seen by the visual system and to weight the visible differences according to the sensitivity of the visual system to those differences. For example, the S-CIELAB model includes a prefilter stage based upon the CSFs of the luminance and chromatic channels.\textsuperscript{16} Similarly, a model of the luminance CSF is used in Daly’s visible-difference predictor.\textsuperscript{9} There is no standard set of CSF measurements or standard CSF functions or models and this has inhibited the standardization of imaging models that utilize the CSF. The chromatic effect on luminance contrast sensitivity that has been modeled in this work is relevant to such imaging models. The use of a CSF for the luminance channel for colour-imaging analysis that was determined using achromatic stimuli would, we argue, overestimate the luminance channel’s sensitivity. Curiously, this chromatic effect may explain informal observations by many observers that black-and-white photographs are more interesting in an artistic sense than coloured counterparts; for the chromatic effect would suggest that the visual system is less sensitive to tonal variation when viewing a coloured image than when it is viewing a chromatically neutral image. The purpose of the work described in this article was to develop a model for the luminance CSF that could be useful for imaging models and that includes the chromatic effect. A model has been presented that has been based upon an existing model published by Barten.\textsuperscript{9} Barten’s model was chosen as the starting point for this work because it is analytical, relatively simple, and can predict those effects that are important for image analysis.