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# Quantitative assessment of commercial filter 'aids' for red-green colour defectives

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**Abstract**

The claims made for 43 commercial filter 'aids', that they improve the colour discrimination of red-green colour defectives, are assessed for protanomaly and deuteranomaly by changes in the colour spacing of traffic signals (European Standard EN 1836:2005) and of the Farnsworth D15 test. Spectral transmittances of the 'aids' are measured and tristimulus values with and without 'aids' are computed using cone fundamentals and the spectral power distributions of either the D15 chips illuminated by CIE Illuminant C or of traffic signals. Chromaticities ($l$,$s$) are presented in cone excitation diagrams for protanomaly and deuteranomaly in terms of the relative excitation of their long (L), medium (M) and short (S) wavelength-sensitive cones. After correcting for non-uniform colour spacing in these diagrams, standard deviations parallel to the $l$ and $s$ axes are computed and enhancement factors $E_l$ and $E_s$ are derived as the ratio of 'aided' to 'unaided' standard deviations. Values of $E_l$ for traffic signals with most 'aids' are <1 and many do not meet the European signal detection standard. A few 'aids' have expansive $E_l$ factors but with inadequate utility: the largest being 1.2 for traffic signals and 1.3 for the D15 colours. Analyses, replicated for 19 'aids' from one manufacturer using 658 Munsell colours inside the D15 locus, yield $E_l$ factors within 1% of those found for the 16 D15 colours.

**Keywords:** colour discrimination, coloured filters, D15 colours, deuteranomaly, protanomaly, traffic signals

## Introduction

The use of coloured filters as aids for colour defectives has been reviewed by Schmidt (1976) and more recently by Sharpe and Jägle (2001). Linhares *et al*. (2008) have reported the effect of coloured filters on normal and dichromatic vision and have derived global estimates of chromatic 'diversity' in CIELAB colour space.

In this paper, we examine the effect of such filters on the spacing of man-made colours for protanomaly and deuteranomaly which together represent some 75% of all red-green colour vision deficiencies. Two sets of test colours were selected: 16 Munsell matte colours of moderate saturation provided by the Farnsworth D15 test (together with a sub-set of 658 Munsell matte colours of equal or lower saturation inside the D15 locus) and three colours of greater saturation provided by the red, yellow and green traffic signal lights specified in a European Standard (CEN, 2007). These sets were chosen to facilitate examination of aspects of utility (D15) and of safety (signals) in filter 'aids'.

It is important in assessing the effectiveness of filter 'aids', intended for use by red-green colour defectives, to separate the attributes of colour associated with 'red-green' and 'blue-yellow' discrimination and to exclude luminance variations from any analysis. Cone excitation chromaticity diagrams provide that possibility. In earlier studies, Moreland and Westland (2002, 2006) used such diagrams to assess the effect of varying amounts of macular pigment on the spacing of colours in normals,

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protanomals (Pa) and deuteranomals (Da). The problem of bias introduced by the large non-uniformity in the diagrams was recognised but not resolved. While several uniform chromaticity diagrams have been developed for normal trichromats (Wyszecki and Stiles, 1982), none exist for Pa or for Da. However, transformations for reducing the non-uniformity of the cone excitation chromaticity diagrams of each of these observers are developed in the Appendix.

## Method

The spectral transmittances of 43 coloured lenses (five manufacturers) were measured for normal incidence. Most of the lenses [19 ChromaGen (ChromaGen Europe, Rotterdam, the Netherlands); 10 ColorMax (ColorMax Technologies Inc, Tustin, CA, USA) and 10 Coloryte (Coloryte Hungary Rt, Szentendre, Hungary)], held in the Optics and Radiometry Laboratory of the University of New South Wales, were measured using a Hitachi U-3410 spectrophotometer (Hitachi High Tech, Tokyo, Japan).

Two others, made available by their wearers, were measured as follows: one X-Chrom lens using a Zeiss DMR21 spectrophotometer (Carl Zeiss, Jena, Germany) at the School of Optometry, University of Waterloo and one A5 ColorView lens using a Minolta CS1000 spectroradiometer (Konica Minolta Sensing Inc., Osaka, Japan) at the School of Design, University of Leeds. Two other examples of models A and B ColorView lenses were estimated, assuming a wavelength range of 400–700 nm, by scanning images of unscaled transmittance graphs from the website http://www.color-view.com/products.php.

The 16 reflectance spectra of the D15 test were measured using a Topcon SR-1 tele-spectroradiometer (Topcon Corporation, Tokyo, Japan) in the Optics and Radiometry Laboratory of the University of New South Wales. The 658 reflectance spectra of the Munsell colour subset were obtained from the web site http://spectral.joensuu.fi/. Spectral power distributions of red, yellow and green traffic signal lights were computed from the spectral luminosity distributions listed in the European Standard for sunglasses (Annex B: CEN, 2007) and interpolated at 5 nm intervals. Chromaticities for the sets of test colours, used for estimating changes in colour spacing, are shown in *Figure 1*.

Using data in the range 400–700 nm at 5 nm intervals, tristimulus values with and without 'aids' are computed by convolving average cone fundamentals for Pa and Da (DeMarco *et al.*, 1992) with the spectral power distributions of either the D15 chips illuminated by CIE Illuminant C (Sc) or of the three traffic signals. Chromaticities are presented in Pa and Da analogues of the MacLeod and Boynton (1979) diagram in terms

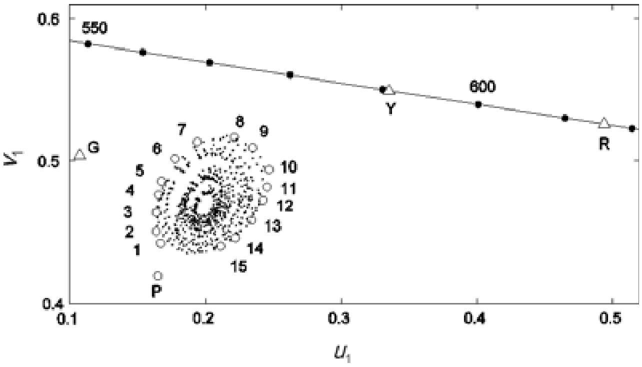


**Figure 1.** Test colours used for analysis in an approximately uniform chromaticity diagram for normal trichromats. Open circles: D15 chips under CIE illuminant C. Points: 658 Munsell colours under illuminant C. Open triangles: red (R), yellow (Y) and green (G) traffic signals. Solid line and small filled circles: spectrum locus.

of the relative excitation of their long (L), medium (M) and short (S) wavelength sensitive cones. After correcting for non-uniform colour spacing in the diagrams for Pa and Da, as described in the Appendix, standard deviations parallel to their L/(L + M) and S/(L + M) axes (designated below by *l* and *s*, respectively) are computed. Enhancement factors $E_l$ and $E_s$ are defined as the ratio of 'aided' to 'unaided' standard deviations. Luminous transmittances of 'aids' for signal lights are computed using the M and L fundamentals respectively as the relative luminous efficiency functions for Pa and Da. All calculations are performed in Excel, with one exception: see Results below.

## Results

An example of the effect of transforms which reduce the non-uniformity of the cone excitation diagrams, as described in the Appendix, on D15 colours and on the calculated values of enhancement factors with and without an 'aid' is illustrated for Pa in *Figure A5*.

$E_l$ and $E_s$ enhancement factors for the D15 colours are shown in *Figure 2* for all 43 aids. More than two-thirds of the 'aids' reduce *l* colour differences ($E_l$ < 1: 29 Pa, 31 Da) and more than a third reduce *s* colour differences ($E_s$ < 1: 17 Pa, 16 Da). We adopt a value of $E \approx 2$ as a feasibly useful enhancement factor for both anomalies. None of the 'aids' achieves this in the *l* direction, where it is needed, but a few (some three or four) do so in the *s* direction, where it is not.

Examples of transmittance spectra for two ChromaGen 'aids' 'Y' and 'P' are shown in *Figure 3* (left). The 'Y' 'aid' slightly expands *l* spacing but highly compresses *s* spacing of D15 colours (Pa and Da: $E_l$ 1.2, $E_s$ 0.1) and the 'P' 'aid' compresses both *l* and *s* spacing ($E_l$: Pa 0.4, Da 0.8. $E_s$: Pa 0.2, Da 0.3). The 'P' 'aid' also has its largest effect on the Yellow traffic signal (*Figure 3*,

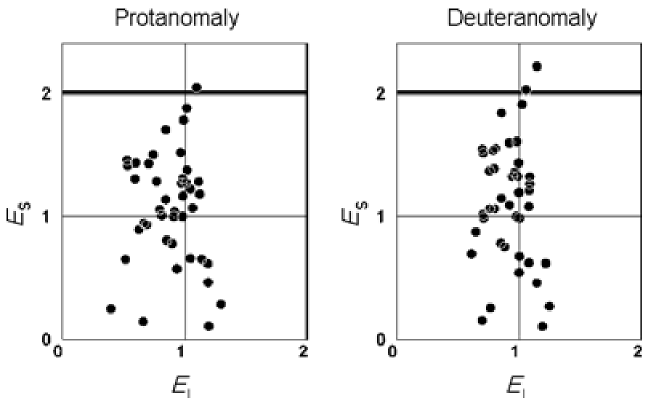


**Figure 2.** $E_l$ ('red-green') and $E_s$ ('blue-yellow') enhancement factors of 43 'aids' for the D15 colour set. Bold horizontal and vertical lines at scale value 2 – utility thresholds.

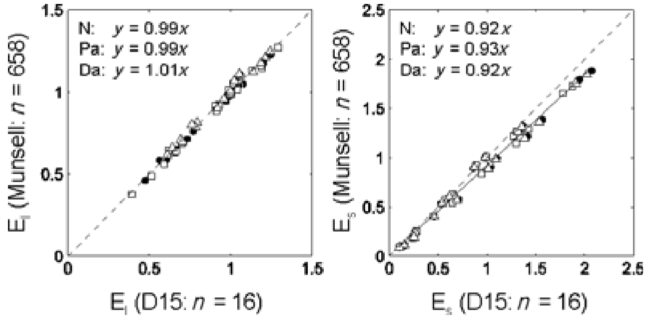


**Figure 4.** Correlation of (left) $E_l$ ('red-green') enhancement factors and (right) $E_s$ ('blue-yellow') enhancement factors, for the D15 colour set and for Munsell colours contained within the D15 locus. Filled circles – normals (N), squares – protanomals (Pa) and triangles – deuteranomals (Da). Broken line: equality. Full line: mean linear regression.

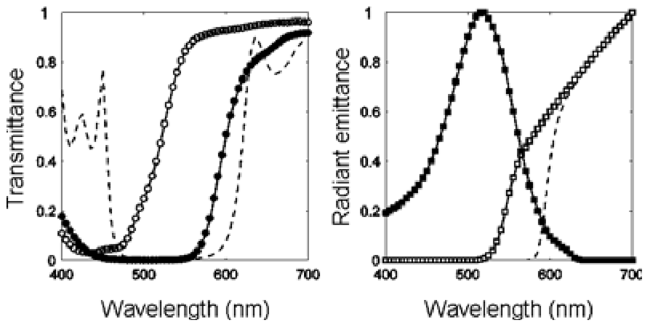


**Figure 3.** (Left) Spectral transmittance of selected 'aids': filled circles – ChromaGen 'Y' contact lens, open circles – ChromaGen 'P' spectacle lens, broken line – ColorMax 'D5' lens. (Right) Normalised spectral radiant emittance of traffic signals: broken line – Red, open squares – Yellow, filled squares – Green.

right). Using the *l,s* chromaticities of Sc and the Yellow signal, with and without 'P', facilitates identification of dominant wavelengths for that signal. However, the signal is so close to the spectrum locus in both instances that the assumption of full chromatic adaptation is hardly necessary. The dominant wavelength shifts down from 583 to 569 nm in Pa and up from 603 to 618 nm in Da. While the latter 15 nm shift may be enough for a category switch from 'yellow' to 'red' for a normal trichromat, it seems less likely for a deuteranomal with a poorer red-green discrimination (Smith *et al.*, 1973).

Is the D15 set of colours representative of a larger population? Computations, using MATLAB (The MathWorks Inc., Natick, MA, USA), repeated for 658 Munsell matte colours contained within an ellipse fitted to chips 1–15 of the D15 test (*Figure 1*), show high correlations ($r^2 \approx 0.99$) between the large Munsell sample and the D15 set for both 'red-green' and 'blue-yellow' axes and for all observers. Scalar differences (*Figure 4*) for all observers are within $\pm 1\%$ for $E_l$, whereas $E_s$ for the former is about 7% or 8% less than for the latter.

The European Standard (CEN, 2007) defines a relative visual attenuation quotient $Q$ for filters in driving and road use by means of two luminous transmittances $\tau$. $Q$ is the quotient $\tau_{sign}/\tau_v$ in which $\tau_v$ applies to the CIE daylight illuminant D65 and $\tau_{sign}$ applies to the spectral power distribution of the traffic signal. $Q$ must not be $< 0.8$ for the Red and Yellow traffic signals, as defined in the Standard, and not $< 0.6$ for the Green. Many 'aids' fail to meet the signal detection standard: $Q$ values being below threshold for the Red (14 Pa, 10 Da), Yellow (27 Pa, 17 Da) or Green (30 Pa and Da) signals. An example of an 'aid', ColorMax 'D5', which would be deemed unsafe in a traffic environment for all three signals ($Q$ values for Red, Yellow and Green respectively: Pa 0.46, 0.12 and 0.02; Da 0.81, 0.40 and 0.03) except for the Red one in Da, is shown in *Figure 3* (left).

The relation between $E_l$ factors and the attenuation quotient $Q$ of all 'aids' for standard traffic signals is shown in *Figure 5*. The $Q$ value plotted for each 'aid' is the one appropriate to the signal most affected (Pa: 10 Red, 33 Green. Da: 10 Red, 1 Yellow, 32 Green). Most 'aids' have compressive $E_l$ factors for traffic signals (34 Pa, 32 Da) and none of the remainder provides a useful ($E_l \approx 2$) enhancement. $E_l$ factors are significantly negatively correlated with least $Q$ for Pa ($r^2 = 0.8$) but not for Da ($r^2 = 0.3$).

The manufacturers ColorMax and Coloryte label their trial sets of coloured lenses as 'P' and 'D' (presumably abbreviations in the sense of Farnsworth's portmanteau terms 'protan' and 'deutan' rather than the more usual dichromatic designations for 'protanopic' and 'deuteranopic'). The ColorView website http://www.color-view.com/products.php refers to 'models A and B' but gives no information that links either of these to the protan or to the deutan category (however, examination of the transmission spectra reveals some

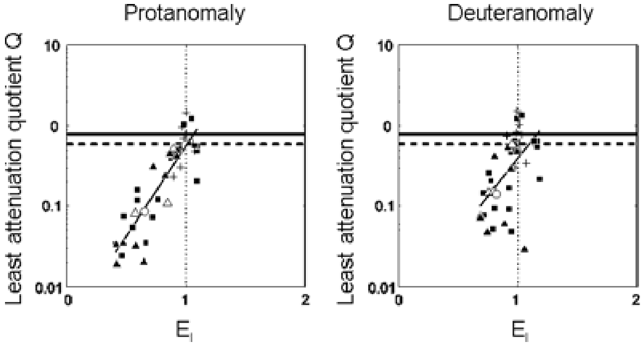


**Figure 5.** $E_l$ ('red-green') enhancement factors and least relative visual attenuation quotient *Q* in protanomaly and deuteranomaly of all 'aids' for traffic signals of the European standard (CEN, 2007). Oblique lines are exponential regressions (note: ordinate scales are logarithmic). Bold horizontal lines: full-safety threshold *Q* = 0.80 for Red and Yellow signals, broken *Q* = 0.60 for Green signal. Bold vertical line- utility threshold at scale value 2. 'Aids': filled squares – ChromaGen, filled triangles – ColorMax, crosses – Coloryte, open triangles – ColorView, open circle – X-Chrome.

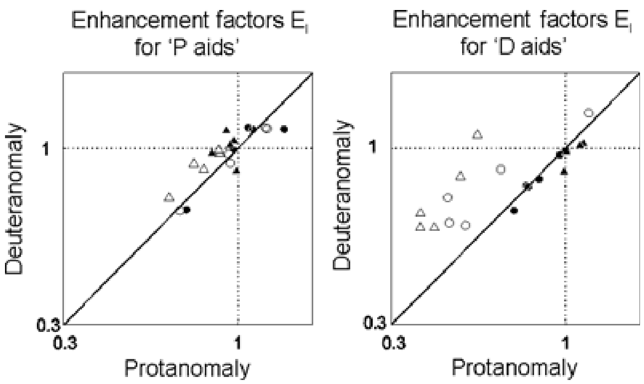


**Figure 6.** $E_l$ factors in protanomaly and deuteranomaly for each of the ColorMax and Coloryte 'P' and 'D' series. Line – equality. Circles – D15. Triangles – Signals. 'Aids': open symbols – ColorMax, filled symbols – Coloryte.

similarity respectively to the 'P' and 'D' series of the now defunct ColorMax company). The $E_l$ factors for Pa and Da are compared for each of the ColorMax and Coloryte 'P' and 'D' series in *Figure 6*. $E_l$ factors of all 'aids' are scattered close to the line of equality for both the 'P' and 'D' series, except that ColorMax 'D aids' perform better for Da than Pa. However, since all the enhancement factors are either less than unity or quite close to it, these differences have no practical utility and indicate that the labels 'P' and 'D' are a poor guide to differential efficacy. This result does not support the claim (Ábrahám, 2001) that Coloryte 'aids' correct the cone sensitivities in anomals to normal.

## Discussion

The European Standard (CEN, 2007) is employed here as a convenient standard. Although it may not reflect modern signalling practice (Dain, 1998) it is likely that such differences would have only a minor effect on the calculations presented here. The blue signal specified in the standard is omitted since it is not a road traffic signal colour. The analyses presented here assume that identical 'aids' are worn binocularly. The effects and problems associated with binocular summation and rivalry when the two eyes are 'aided' differentially have been reviewed by Sharpe and Jägle (2001). Knoblauch and McMahon (1995) have reported experiments in which the binocular use of two different filters, passing long- and middle-wavelength regions of the spectrum, conferred an additional colour dimension to red-green dichromats but they also reported that wavelength discrimination remained quite poor in the Rayleigh region. While classical (2°) red-green dichromats could not obtain any red-green benefit at all from the binocular use of identical filters of any sort, it could be surmised that those possessing extrafoveal anomalous cones (Moreland, 1997) may do so for wide fields of view.

Coloured filters impose luminance changes which may break the camouflage of some colour vision tests (Sharpe and Jägle, 2001) like the Ishihara pseudoisochromatic plates, compromising their design, but not others, such as lantern tests (Swarbrick *et al*., 2001). This paper is not concerned with the effect of 'aids' on the performance of Pa or Da in colour vision tests: the Farnsworth D15 test colours were chosen here merely as a set representing a moderate level of saturation. Unlike Linhares *et al*. (2008), we exclude luminance variations from our analysis, by dealing with projections onto a single chromaticity plane. We use the cone fundamentals of DeMarco *et al*. (1992) which represent Pa and Da averages. The recent identification of subgroups within each Pa and Da category (Stockman *et al*., 2000) indicates possibilities for refining the model. However, the differences found in 'aid' performance between the Pa and Da average classes (*Figures 2 and 5*) are not so large that further refinement would seem profitable.

## Conclusions

None of the 'aids' assessed here for protanomaly and deuteranomaly provide a useful colour enhancement in the 'red-green' direction either for the D15 colours, or for the European Standard (CEN, 2007) red, yellow and green traffic signals; and many reduce such colour differences. Commercial claims made for improved 'red-green' colour discrimination are not upheld. Although some 'aids' do increase 'blue-yellow' colour differences, it is superfluous since discrimination in protanomaly and deuteranomaly for such colours is not impaired. Many 'aids' attenuate traffic signals below the detection threshold set by the European Standard and should be considered as dangerous in a traffic environment.

## Acknowledgements

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## Appendix

We begin with a consideration of normal trichromacy. Adopting the Vos (1978) revision of the CIE 1931 2° colour matching functions, we construct an approximately uniform chromaticity scale diagram using the CIE (1978) transformation equations. Use of these equations is justified since the revision has only a small effect on the gamut of the spectrum locus. But, importantly, the revision also provides a more precise convergence of tritanopic metamers (derived in the manner of Thomson and Wright, 1953 from the 2° cone fundamentals listed at http://www.cvrl.org/). Since the space is revised, we replace the CIE terms $u',v'$ with $u_1,v_1$.

We construct a square grid in the $u_1,v_1$ diagram (*Figure A1*, left) with the pilot line (row 9) set close and parallel to the spectrum locus between 665–685 nm, a region unaffected by S cones. The grid covers most of the D15 chromaticity gamut imposed by the 43 filter 'aids' and the grid line intersections define a set of 86 target chromaticities. Exact spectra for those are computed by combining the spectra of three near-neighbour man-made colours, under Sc, selected from a population of 1687 (1269 Munsell matte papers: spectral reflectance as listed in the University of Joensuu website http://spectral.joensuu.fi/; 256 Lee filters (Lee Filters, Andover, Hants, UK): spectral transmittance data provided by Jon Young (2008, personal communication); and 162 Wratten filters: spectral transmittance as listed in the Kodak catalogue (Eastman Kodak, 1973) and interpolated at 1 nm intervals). The same 86 spectra are then used to construct an equivalent grid in a cone excitation diagram (*Figure A1*, right) using the normal L, M and S fundamentals of DeMarco *et al.* (1992).

Regressions along the grid rows and columns show that linearity is highly conserved (averaged $r^2$ are: for rows 0–9, 0.997 and, for columns A–J, 0.9999), indicating a close correspondence between the Vos (1978) and DeMarco *et al.* (1992) normal colour matching functions. Extrapolation of the row and column regressions converge in a way indicating a projective transform of the $u_1,v_1$ diagram (*Figure A2*). We nominate the constant $l$ and $s$ lines through Sc as reference axes. Regression intersections with the constant $s$ line (defining an $l_p$ scale by regressing $l$ on $s$ for best precision) and the constant $l$ line (defining an $s_p$ scale by regressing $s$ on $l$ for best precision) leads to a rectangular coordinate system (illustrated in *Figure A5* for Pa) as a first step to improve uniformity.

Equivalent grids are constructed in the cone excitation diagrams for Pa and Da (*Figure A3*) using the appropriate anomalous La and Ma fundamentals, respectively, of DeMarco *et al.* (1992). Regressions show that linearity is reasonably conserved (average $r^2$ are, for Pa and Da respectively, rows 0–9, 0.97 and 0.92 and for columns A–J, 0.83 and 0.90). All regressions converge in a way suggesting that these diagrams may be treated as approximate projective transforms but the systematic 'folding' of 'aberrant' regression lines around the convergence region suggests caution in that regard. The question then arises: can the regression intersections with reference $s$ and $l$ lines through Sc also be used to improve uniformity for these two anomalies in the same way as for normals?

The projection scales (denoted by subscript '$p$') thus created are compared in *Figure A4* (top) with an ideal uniform scale. Quantitative differences between normal and anomalous $l_p$ scales are large and so, to facilitate comparison, relative scales are used. The figure, while confirming the large non-uniformity of the $s_p$ scales, reveals a lesser non-uniformity for the $l_p$ scales. There are qualitative differences between the three observers for the two sets of scales but these are not large. Using the actual scale spacing around the Sc point on the constant $l$ and $s$ lines as a seed, we form idealised target uniform scales and use Excel Solver to optimise compressive logarithmic functions which transform each actual scale to match them closely: *Figure A4* (bottom).

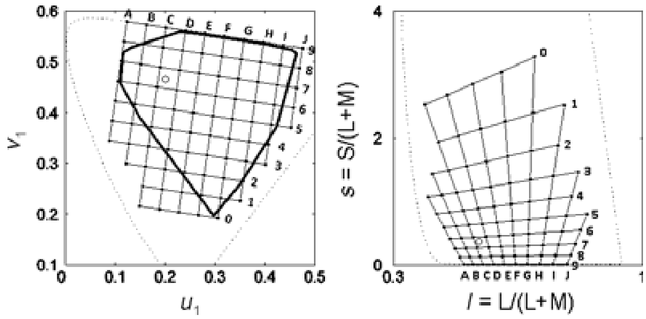



**Figure A1.** (Left) A square grid, in an approximately uniform chromaticity diagram, constructed to cover most of the D15 colour gamut (bold locus) imposed by 43 filter 'aids'. Circles at the grid line intersections are chromaticities for 86 hybrid spectra. (Right) Chromaticities for the same 86 spectra define an equivalent grid in a cone excitation diagram using the normal L, M and S fundamentals of DeMarco *et al.* (1992). Both graphs: Broken line – spectrum locus. Open circle – CIE illuminant C. Alphabetic and numeric labels designate respectively the columns and rows referred to in the text.

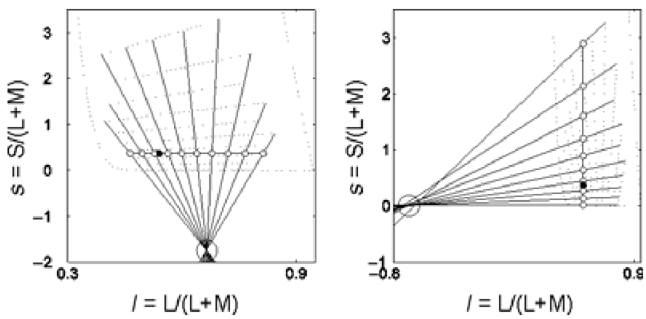



**Figure A2.** Bold lines: extrapolations of the column and row regressions for normal trichromats in *Figure A1* (right) converge near the centres of the large open circles. Horizontal (constant $s$) and vertical (constant $l$) lines through the CIE illuminant C point (filled circle) are intersected (open circles) defining new $l_p$ and $s_p$ scales as a first step to improve uniformity.

**Table A1.** Regression convergence points for projective transforms

| Reference $l$ and $s$ lines pass through the CIE Sc point | For the $l$ line | | For the $s$ line | |
|---|---|---|---|---|
| | $l_s$ | $s_s$ | $l_l$ | $s_l$ |
| Normal | 0.6655 | −1.7559 | −0.69 | 0 |
| Protanomal | 0.5894 | −2.2270 | 0.40 | 0 |
| Deuteranomal | 0.5200 | −1.1146 | 1.18 | 0 |

**Table A2.** Coefficients of compressive transforms (applied after projective transforms)

| Scale | Observer | a | b | c | d | |
|---|---|---|---|---|---|---|
| $l_p$ | Normal | 1.37663 | 1.12581 | 0.11808 | 0.82820 | $l_{cp} = a.\log([l_p]^b + c) + d$ |
| | Protanomal | 0.40845 | 0.00819 | 0.99169 | 1.55346 | |
| | Deuteranomal | 0.22902 | 0.99822 | 0.41226 | 0.74154 | |
| $s_p$ | Normal | 1.14986 | 1.30215 | 0.19813 | 0.74494 | $s_{cp} = a.\log([s_p]^b + c) + d$ |
| | Protanomal | 0.99573 | 1.34513 | 0.14196 | 0.76127 | |
| | Deuteranomal | 1.01368 | 1.32161 | 0.16052 | 0.74242 | |

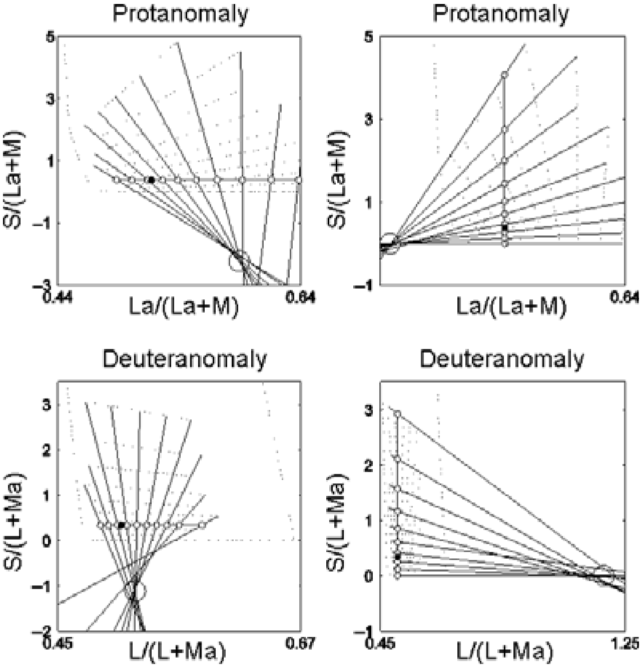



**Figure A3.** Definition of new $l_p$ and $s_p$ scales for protanomaly and for deuteranomaly. Other details as *Figure A2*.

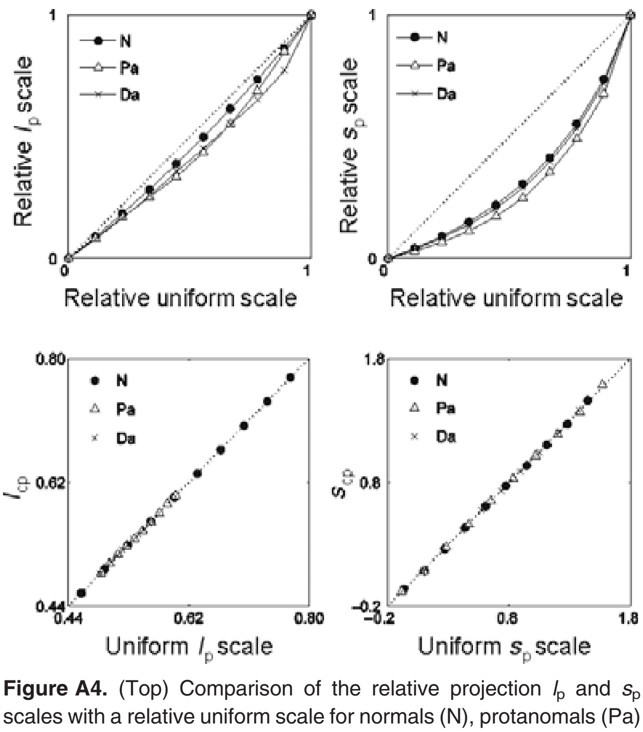
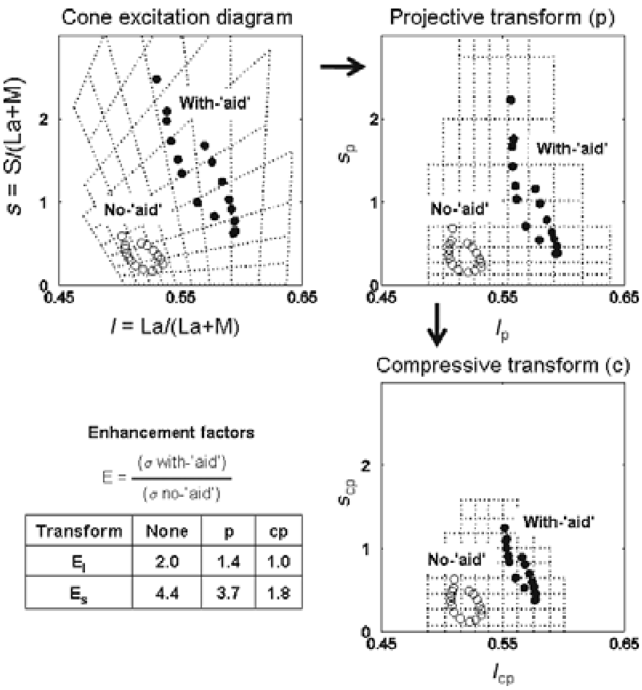



**Figure A4.** (Top) Comparison of the relative projection $l_p$ and $s_p$ scales with a relative uniform scale for normals (N), protanomals (Pa) and deuteranomals (Da). (Bottom) compressive transforms $l_{cp}$ and $s_{cp}$ of actual scales compared with uniform scales based on the actual $l_p$ and $s_p$ spacing around the CIE illuminant C point.




| Transform | None | p | cp |
|---|---|---|---|
| $E_l$ | 2.0 | 1.4 | 1.0 |
| $E_s$ | 4.4 | 3.7 | 1.8 |

**Figure A5.** Effect of the projective and compressive transforms on D15 colours for protanomaly with and without a ChromaGen 'V aid'. Broken lines – row and column regressions as in *Figure A3*. Circles – D15 colours. (Top left) cone excitation diagram. (Top right) projective transform. (Bottom right) compressive transform. (Bottom left) table lists estimates of enhancement factors $E_l$ ('red-green') and $E_s$ ('blue-yellow') at each stage. Note: the ordinate and abscissa scales are chosen for clarity, not fidelity.

The convergence points adopted for projecting any point onto the constant $l$ and $s$ lines are listed in *Table A1*. Projections $l_p$ and $s_p$ of any target chromaticity $l,s$ onto the respective constant $s$ and $l$ lines are given by

$$l_p = l - (l - l_s)(s - s_k)/(s - s_s)$$

$$s_p = s(l_k - l_l)/(l - l_l)$$

where $l_k,s_k$ is the Sc illuminant point and $l_s,s_s$ and $l_l,0$ are the respective convergence points for column and row regressions.

The convergence points $l_s,s_s$ for column regressions on the constant $l$ line for Pa and Da, listed in *Table A1*, are means of the cluster of 10 closely grouped intersections for the five regressions of columns C–G (*Figures A2 and A3*, left). Aberrant regressions are dealt with separately by using the regression intersections for columns A,B; B,C; G,H; H,I and I,J as alternative convergence points (the effect on $E_l$ factors of this separate treatment is quite small).

The convergence points $l_l,0$ for row regressions on the constant $s$ line for Pa and Da are set at $s = 0$, and $l_l$ is selected by visual inspection to satisfy the topmost row regressions (0–2 in *Figures A2 and A3*, right): these being the most critical due to their greater slopes.

The coefficients for the compressive logarithmic functions which transform the $l_p$ and $s_p$ scales to the more uniform $l_{cp}$ and $s_{cp}$ scales are listed in *Table A2*. No attempt has been made here to harmonise the $l_{cp}$ and $s_{cp}$ scales. Harmonisation, based on the scale spacing around the Sc point, would require the $s_{cp}$ scales to be reduced by a factor of about five for normals and probably by similar amounts for Pa and Da. The projective transforms, by their geometric definition, do not affect the Sc point position but the compressive transforms do shift it slightly ($< 0.0015$) for all abscissae and negligibly ($< 0.0000004$) for all ordinates. It could be argued that, in view of the complex relation between colour discrimination and viewing parameters (Miyahara *et al*., 1993), correcting the moderate non-uniformity of the $l_p$ scales by compressive transforms is unwarranted. However, the corrections are retained here for completeness.

An example of the effect of transforms on D15 colours and on the calculated values of enhancement factors with and without an 'aid' is illustrated in *Figure A5* for protanomaly and a ChromaGen 'V' contact lens. The transforms reduce the misleadingly large enhancement factors derived from the non-uniform cone excitation diagram by half for *l* and by more than half for *s*.