Efficient representation of spectral transmittance

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ABSTRACT

The spectral characteristics and the nature of the basis functions computed for a linear model to represent the spectra are dependent upon the data sets that are used[3]. One set of Munsell reflectance and one set of transmittance of glass samples were examined by applying linear modelling. The results show differences between the two data sets in terms of basis functions and reconstructions errors. The basis functions from the Munsell set provided adequate representations, at least colorimetrically, of the glass transmittance samples despite the difference in colour gamuts.

1. INTRODUCTION

Recent computational models of colour vision demonstrate that it is possible to achieve exact colour constancy if the range of lights and objects observed is limited to those described by a low-dimensional linear model. The statistics of the reflectance spectra of objects have been analysed by many researchers[1,2]. Some of this work has suggested that both the spectral characteristics themselves and the nature of the basis functions computed for a linear model to represent the spectra are dependent upon the data sets used[3]. Reflectance spectra investigated for different types of objects have included oil paint, human skin, and printed surfaces. Few studies have been carried out using transmittance spectra, however. Reflectance and transmittance spectra share some common physical processes[4] that underlie their properties, but scattering of light generally plays a more prominent role in the reflectance process, resulting in a greater diffuse component and lower maximum density. Therefore it is an interesting question whether similar linear models could be used to represent both types of spectra. In this study the question has been addressed using transmittance spectra from coloured glass typically used in the restoration of stained glass windows.

2. METHOD

Any spectral distribution can be approximated to a specified degree of accuracy as a weighted sum of basis functions. Thus, a spectrum $P(\lambda)$ can be described by:

$$P(\lambda) = a_1 P_1(\lambda) + a_2 P_2(\lambda) + \cdots + a_n P_n(\lambda) \tag{1}$$

where $P_i(\lambda)$ is the i^{th} basis function and a_i is the coefficient for the i^{th} basis function. Once the $P_i(\lambda)$ are known, a set of n weights a_i is sufficient to specify any spectrum in the model.

A set of 245 samples of stained glass was measured by a transmission spectrophotometer at intervals of 10 nm in the range 400-700 nm[5]. The aim was to investigate whether any differences in the basis functions computed for transmittance and reflectance were caused by spectral (rather than colorimetric) differences between this set and a similar set of reflectance samples. The reflectance spectra for the latter were selected from a larger set of 1269 Munsell reflectance spectra to give an approximate chromatic match to each of the 245 transmittance spectra. The CIE *XYZ* values of all the transmittance spectra and Munsell reflectance spectra were calculated for illuminant D65 and, for each transmittance spectrum, the Munsell reflectance closest in *XYZ* space was chosen to be the best colorimetric match. In order to justify the matching technique, the difference in *XYZ* space was calculated for each matched pair, giving the statistics in Table 1.

Table 1 Statistics of distance in *XYZ* space of trying to match tristimulus value of the stained glass from the 1269 Munsell samples.

Distance	mean	max	min	std
XYZ	4.3824	32.1814	0.1502	4.29

From Table 1 it is evident that the match was not very close for all the pairs. In Figure 1 the CIE chromaticity coordinates are plotted for both sets of data and it can be seen that the gamut of the glass samples was much larger than that of the reflective samples. It is for this reason that the figures in Table 1 are quite large. Indeed, this was not just a problem with the particular set of reflectance data chosen – the Munsell set – since almost all reflectance spectra have limited chromaticity compared with many of the transmittance samples in Figure 1. Thus although we tried to match each spectrum in the transmittance set with a chromatically identical Munsell sample, this was achieved with only a limited set. Nevertheless, we argue that the set of Munsell samples chosen based on this technique for the comparison that follows was more appropriate than an arbitrary subset of the Munsell samples. Two sets of basis functions were derived, one from each of the two data sets. For each data set, reconstructed spectra were generated using their own set of basis functions and using the other set of basis functions and the reconstruction errors were calculated

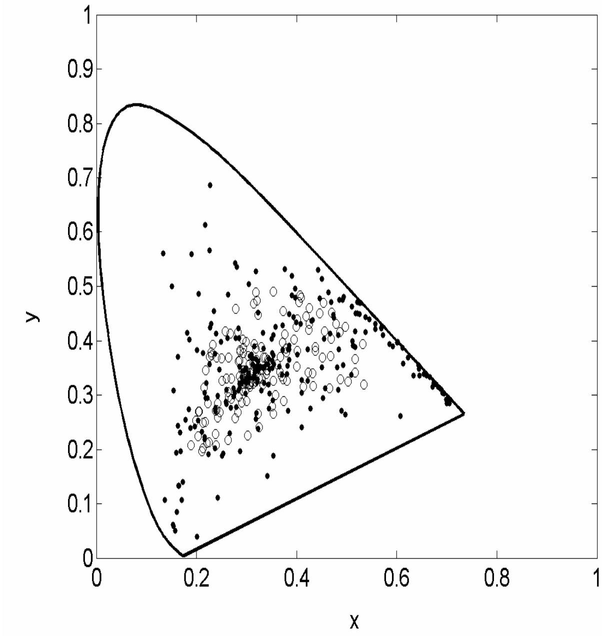


Figure 1: Chromaticity of glass (dots) and Munsell (circles) samples

3. RESULTS AND CONCLUSIONS

Figure 2 shows the mean spectral curves for both data sets. It is evident that both were smooth functions that were band limited[6] in spectral-frequency space. The main difference is observed at the very short and very long wavelengths, where the absorption of glass typically drops away[7]. Tables 2 and 3 show the reconstruction error in RMS and CIELAB colour difference respectively for the two data sets, computed from each of the two sets of basis functions. Generally for both sets of samples, and irrespective of which basis functions were used, the reconstruction errors fell with an increase in the number of basis functions. Fewer basis functions were needed for the Munsell data, however, to achieve a specified degree of accuracy. For example, Table 2 shows that the mean RMS error approached a value of 1% with only 4 basis functions for the Munsell spectra, whereas 6 basis functions were required for the glass. Similar results are seen in Table 3 for the ΔE^* error approaching the nominal visibility threshold of 1. It is clear that both spectrally and colorimetrically, no matter whether the basis functions were derived from the glass or Munsell samples, the reconstruction errors for the glass samples were always greater, most likely because of their greater gamut. For the Munsell data, smaller reconstruction errors in CIELAB ΔE^* resulted when the basis functions were derived from the Munsell samples rather than from the glass samples. The fitting of the Munsell basis functions to the glass samples was better colorimetrically than spectrally. This may have happened because the colour-matching functions have low values at very short and very long wavelengths, where the spectral error in the reconstructions is large. From these results it seems that, at least colorimetrically, the basis functions from the Munsell set provide adequate representations of the glass transmittance samples despite the difference in gamuts.

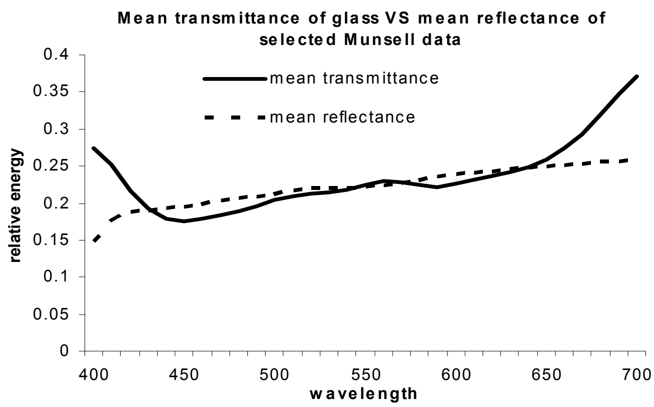


Figure 2: Mean transmittance of glass vs. mean reflectance of selected Munsell data.

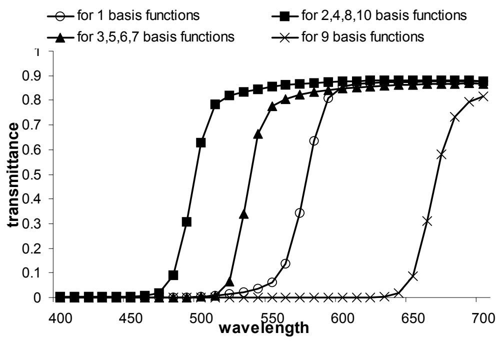
Table 2: Reconstruction errors in RMS for a linear model with varying number of parameters. "G" means that the basis functions were derived from the stained glass samples. "M" means that the basis functions were derived from the selected Munsell data set.

Number of basis functions	**Mean RMS**				**Maximum RMS**			
	Glass Samples		**Munsell Samples**		**Glass Samples**		**Munsell Samples**	
	G	M	G	M	G	M	G	M
1	0.1293	0.1325	0.0809	0.0762	0.3645	0.371	0.2453	0.2543
2	0.0769	0.0858	0.0497	0.0379	0.2432	0.2378	0.1675	0.1557
3	0.0445	0.0637	0.0354	0.0174	0.1540	0.1942	0.1046	0.0983
4	0.0335	0.0542	0.0282	0.0120	0.1148	0.1632	0.0875	0.0723
5	0.0200	0.0442	0.0141	0.0093	0.1073	0.1111	0.0539	0.0318
6	0.0132	0.0280	0.0114	0.0072	0.0877	0.1075	0.0500	0.0311
7	0.0098	0.0168	0.0090	0.0051	0.0633	0.0724	0.0369	0.0232
8	0.0075	0.0139	0.0070	0.0039	0.0538	0.0681	0.0206	0.0199
9	0.0057	0.0125	0.0057	0.0028	0.0344	0.0644	0.0201	0.0101
10	0.0048	0.0116	0.0052	0.0020	0.0262	0.0593	0.0195	0.0067

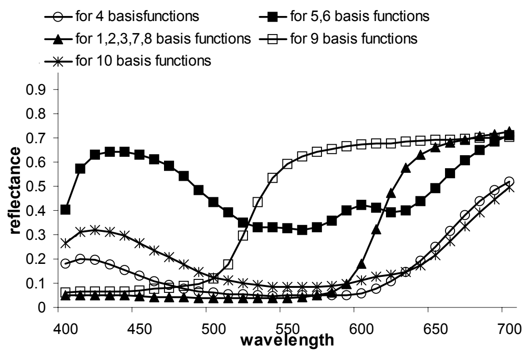
Table 3: Reconstruction errors in CIELAB ΔE^* for a linear model with varying number of parameters. "G" means the basis functions were derived from the stained glass samples. "M" means that the basis functions were derived from the selected Munsell data set.

Number of basis functions	**Mean CIELAB ΔE^***				**Maximum CIELAB ΔE^***			
	Glass Samples		**Munsell Samples**		**Glass Samples**		**Munsell Samples**	
	G	M	G	M	G	M	G	M
1	36.73	36.64	23.54	23.44	124.84	126.41	69.32	70.80
2	32.55	33.45	16.35	15.81	178.38	172.36	108.19	112.39
3	20.68	13.28	9.97	3.46	137.41	70.36	35.26	25.65
4	18.04	9.23	8.95	1.31	116.62	45.92	31.04	12.29
5	6.13	6.75	1.87	0.86	71.03	24.87	12.04	4.76
6	1.22	3.46	0.52	0.36	20.73	48.97	3.78	1.85
7	0.83	1.39	0.36	0.21	11.00	12.27	2.28	1.07
8	0.45	1.44	0.26	0.20	5.81	13.24	1.70	1.33
9	0.46	0.96	0.26	0.15	5.67	11.66	1.73	0.87
10	0.44	1.21	0.25	0.08	5.76	17.20	1.69	0.45

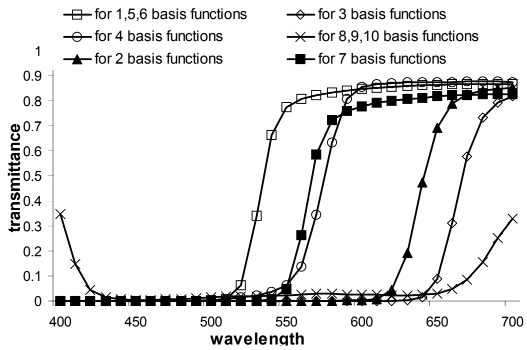
It is also apparent from Tables 2 and 3 that the maximum reconstruction errors were much larger than the mean errors. In Figure 3 we plot the spectra which caused the largest RMS and CIELAB ΔE^* errors for various numbers (1 to 10) of basis functions. Here we consider only the basis functions derived from their own data set.



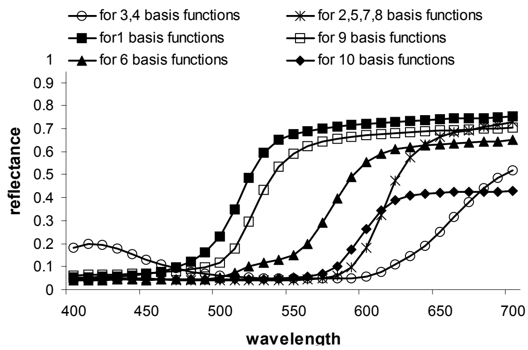
(a) The glass samples presenting the largest RMS errors.



(b) The Munsell samples presenting the largest RMS errors.



(c) The glass samples presenting the largest CIELAB ΔE^* errors.



(d) The Munsell samples presenting the largest CIELAB ΔE^* errors.

Figure 3: The samples which give the largest reconstruction error when fitting with their 'own' set of basis functions in term of RMS error and CIELAB colour difference.

It is not surprising that most of the spectra that caused large reconstruction errors had relatively high spectral frequencies, characterised by high slopes in the transition from absorbing to transmitting. The glass samples were thus more saturated than the Munsell samples, which led to larger reconstruction errors. One particular transmittance spectrum for a deep violet glass produced the largest reconstruction error when 8,9,10 basis functions were used. As can be seen from Figure 3(c), it had a spectrum with very low values in the middle but rising at the two ends. Such spectra are not found in the Munsell reflectance data and are thus not well represented by the Munsell basis functions.

References

1. M. Thomson and S. Westland, "Spectral colour statistics of surfaces", in *Colour Imaging Science*, ed. L.W. MacDonald and M.R. Luo, John Wiley, 2002.
2. P.J. Pakinnen, M. Hauta-Kasari, A. Kaarna, J. Lehtonen, P. Koponen, and T. Jaaskelainen, "Multispectral Image Compression", *Proc. 2nd Intl. Symp. on Multispectral Imaging*, Chiba University, Japan, 49-58, 2000.
3. J.Y. Hardeberg, "On the spectral dimensionality of object colours", *Proc. IS&T First European Conference on Colour in Graphics, Image and Vision (CGIV)*, 480-485, 2002.
4. R.J.D. Tilley, *Colour and optical properties of materials*, John Wiley, 2000.
5. T. Song, L.W. MacDonald, P. Rudolf and K.D.M. Findlater, "Prototype of Image Processing Software, Annex 1 - Characterisation of Digital Camera", EC-FP5 project IST-2000-28008 VITRA, Deliverable Report 12, August 2004.
6. L.T. Maloney, "Evaluation of linear models of surface spectral reflectance with small numbers of parameters", *Journal of the Optical Society of America A*, **3** (10), 1673-1683, 1986.
7. W.A. Weyl, *Coloured Glasses*, Society of Glass Technology, Sheffield UK, 1951.